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NARODOWA STRATEGIA SPÓJNOŚCI

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„SIGNAL PROCESSING”

Prezentacja multimedialna współfinansowana przez Unię Europejską w ramach Europejskiego Funduszu Społecznego w projekcie pt.

„Innowacyjna dydaktyka bez ograniczeń - zintegrowany rozwój Politechniki Łódzkiej - zarządzanie Uczelnią, nowoczesna oferta edukacyjna i wzmacniania zdolności do zatrudniania osób niepełnosprawnych”

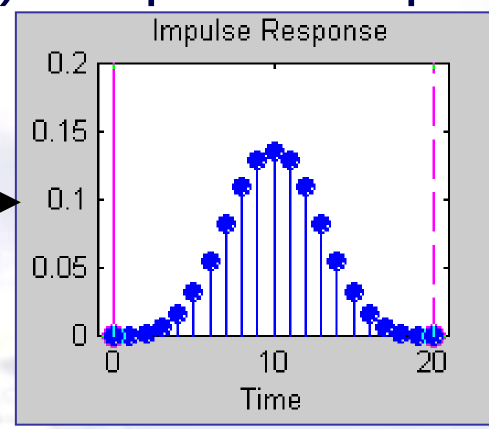
Digital filters

Digital signal processors (DSP), programmable circuits

$x(n) \rightarrow$ Digital filter $\rightarrow y(n)$

$h(n)$ - impulse response

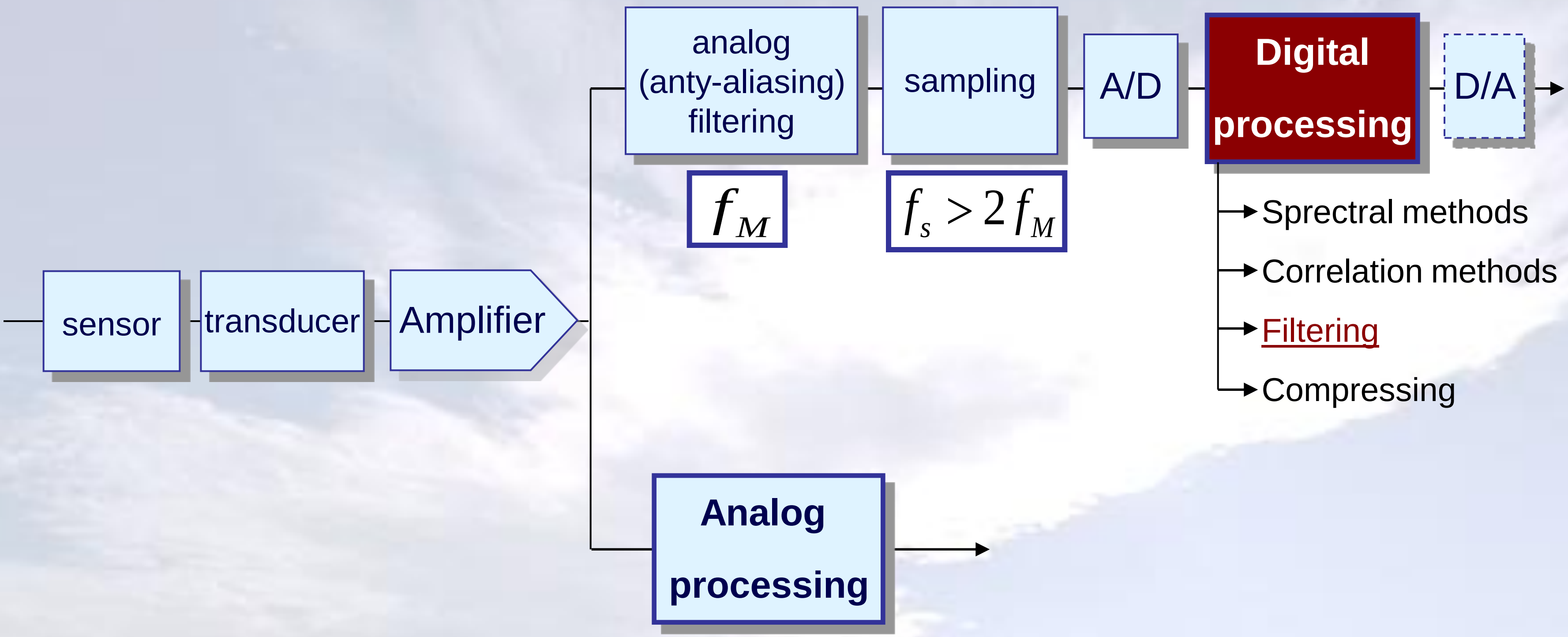
$x(n) \rightarrow$



$\rightarrow y(n)$

$$y(n) = x(n) * h(n)$$

Signal processing



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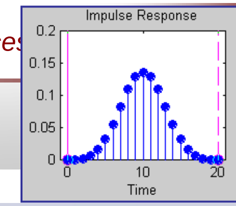

Digital filters

Why to filter the signal?

- to reduce noise (eg. from the mains network)
- to change the spectral characteristic of the signal (*preemphasis, deemphasis*)
- to filter out the components of intrest from the given signal (detection)

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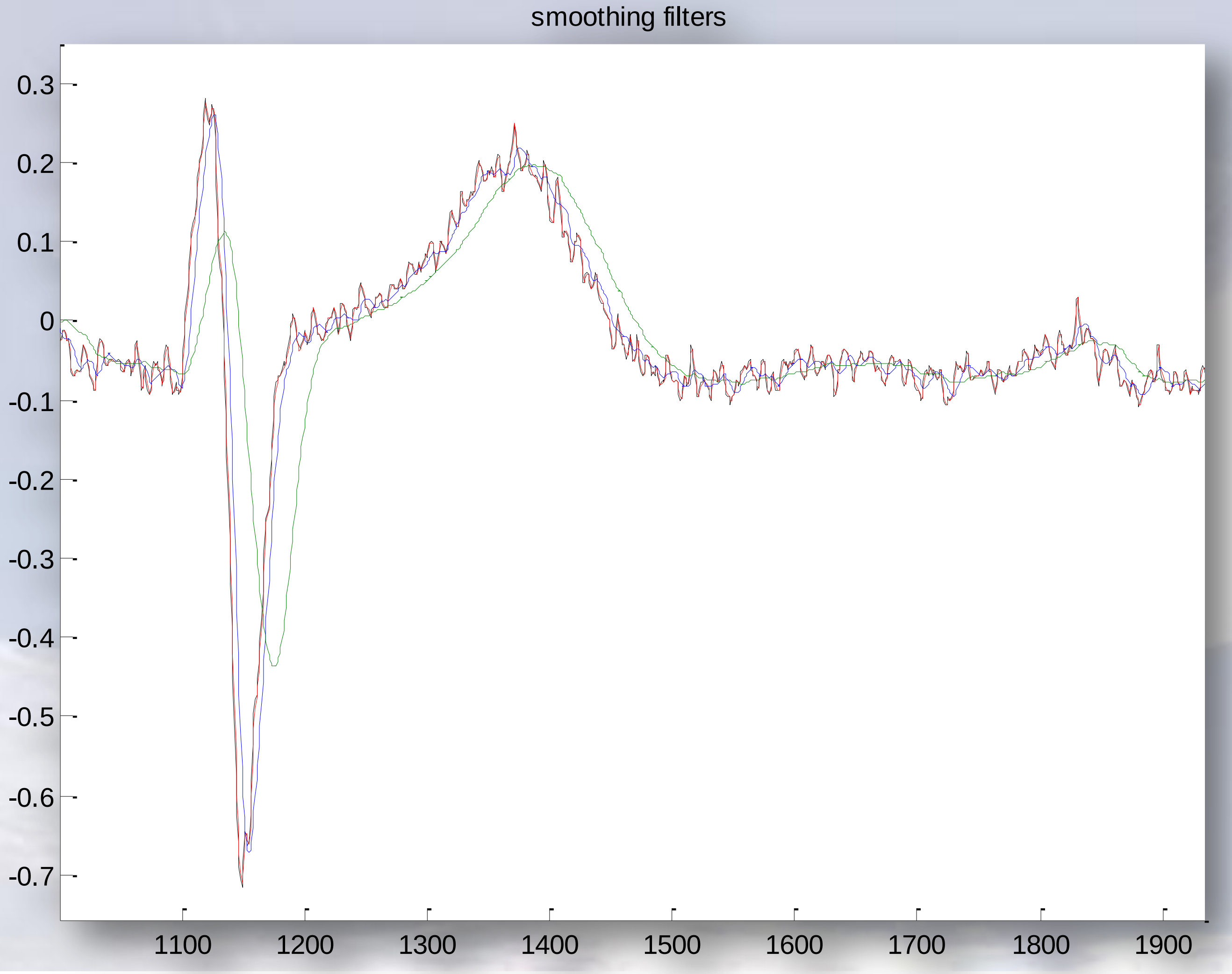
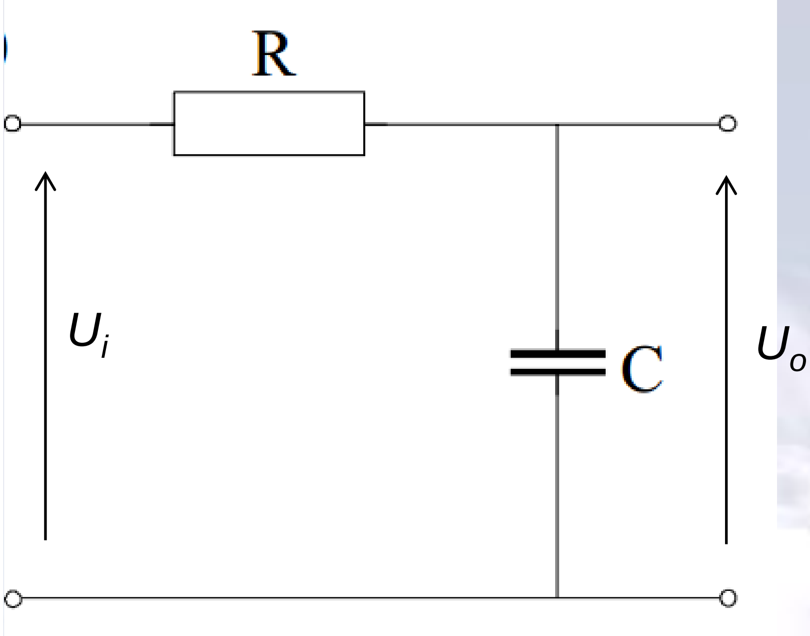
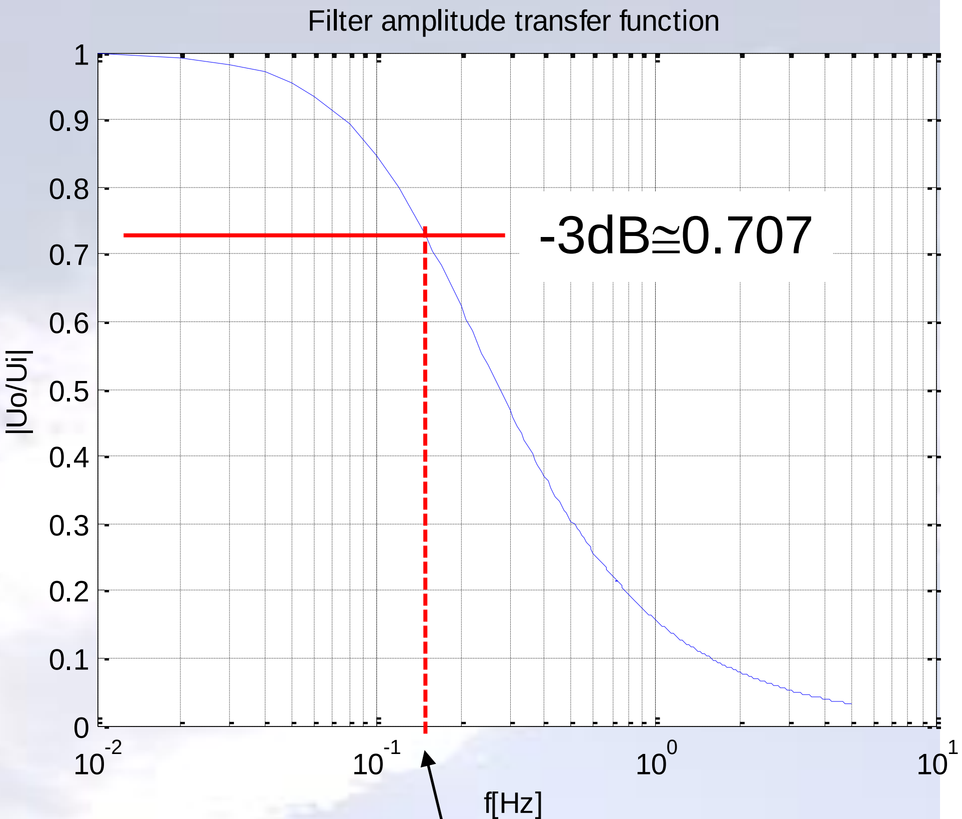
Signal smoothing by filtering



An analogue low-pass filter example



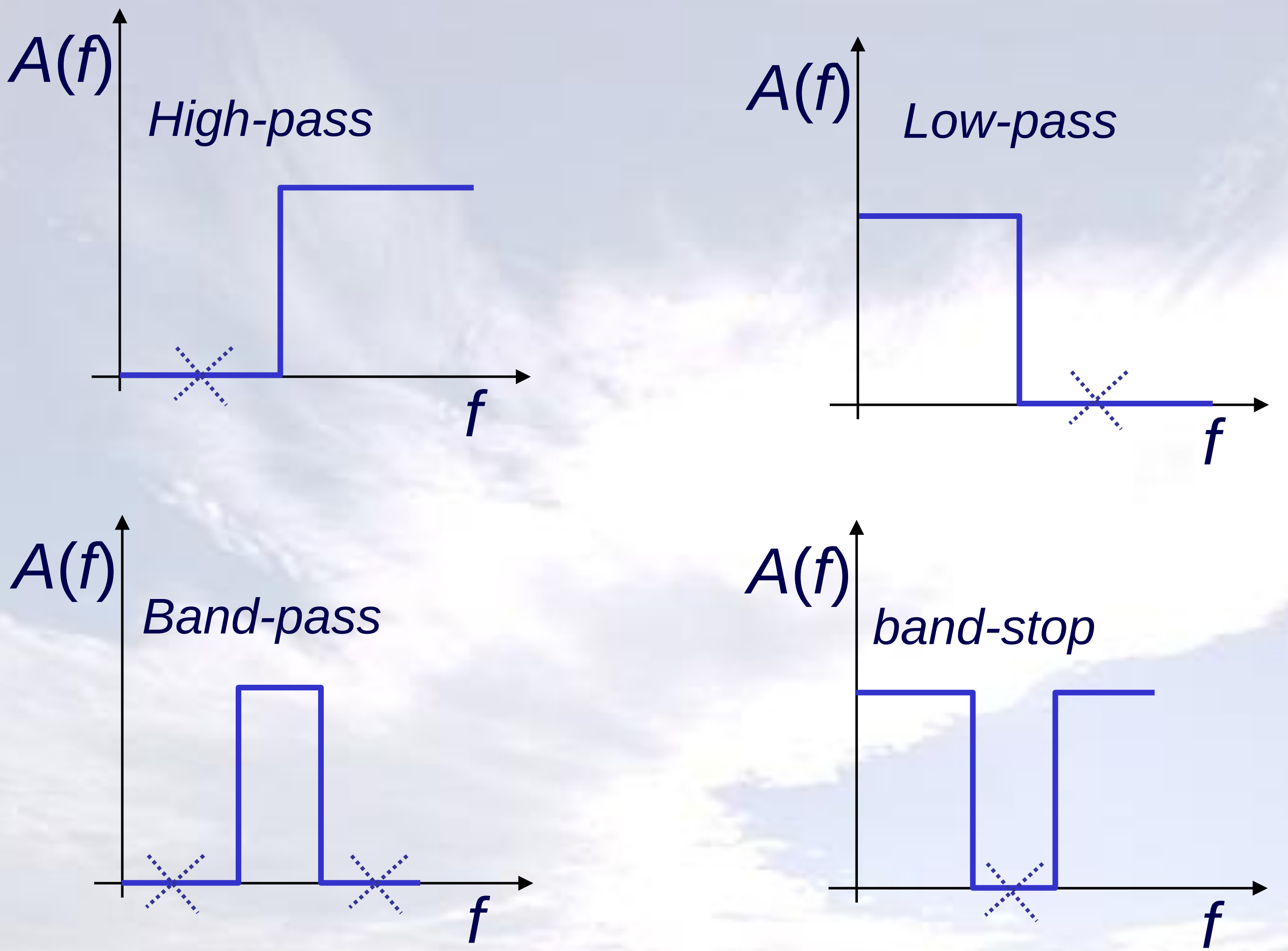


Amplitude transfer function:

$$\left|\frac{U_o}{U_i}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$f_c = ?$

Frequency characteristics – ideal filters



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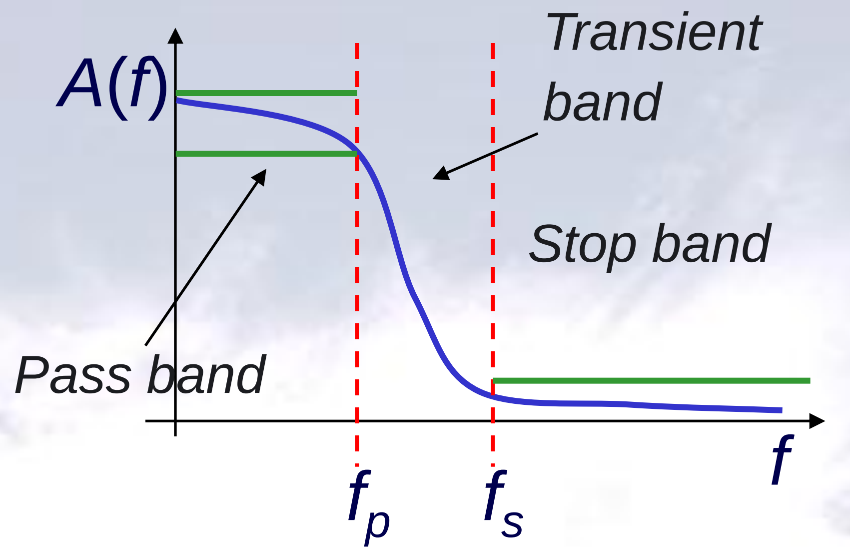
Frequency characteristics

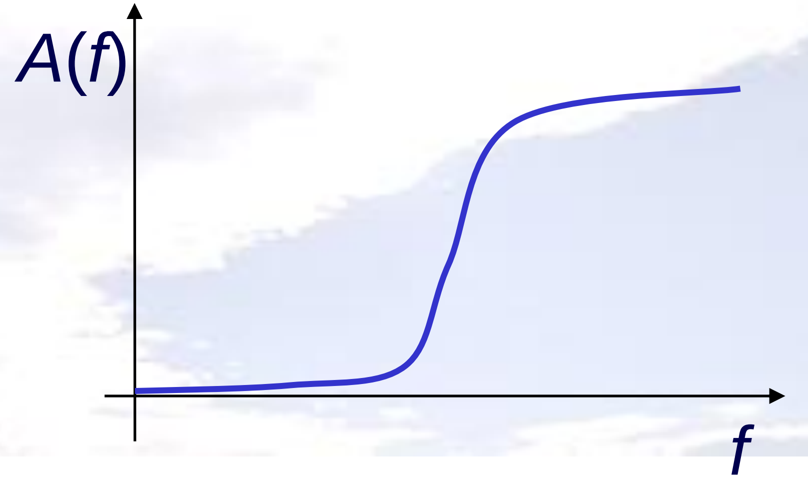
Low-pass filter

(eg. *antialiasing filter, noise reduction*)



High-pass filter

(eg. *preemphasis*)



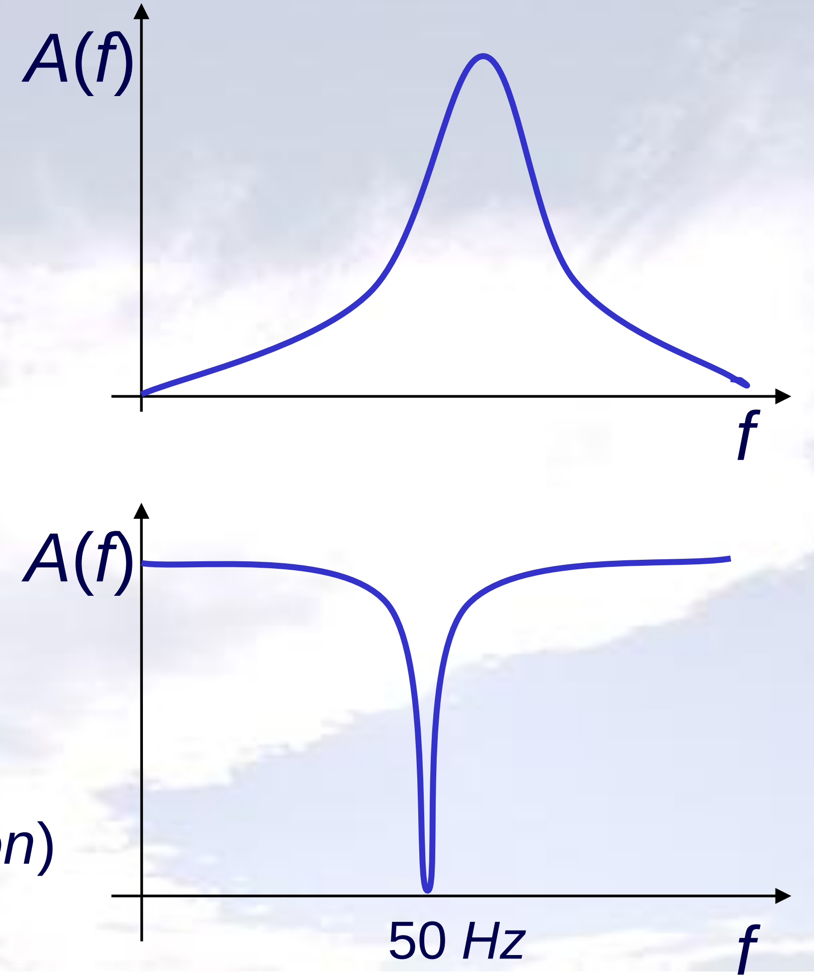
Frequency characteristics

Band-pass filter

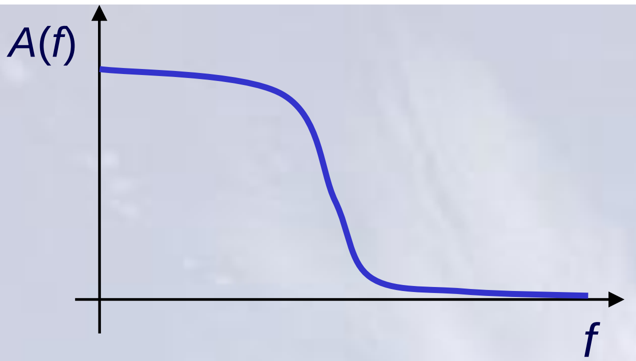
(eg. *feature detection*)

Band-stop fiter

(eg. *mains interference reduction*)

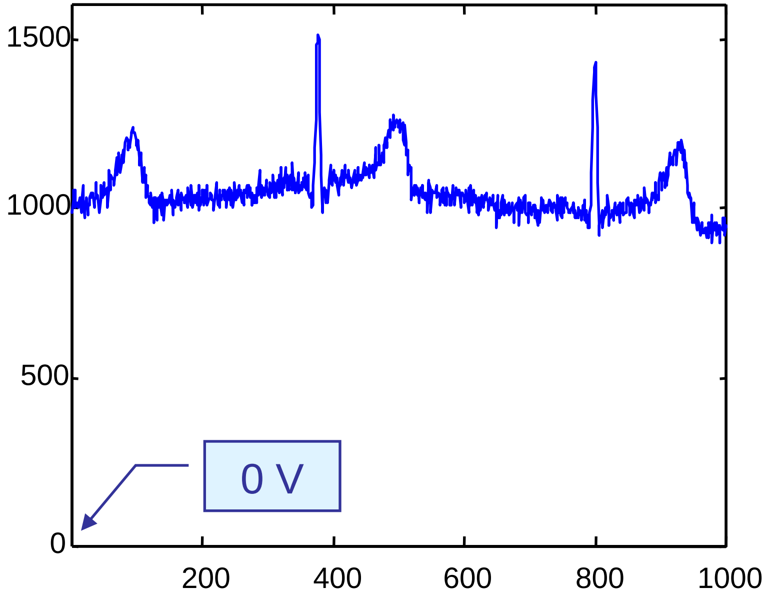


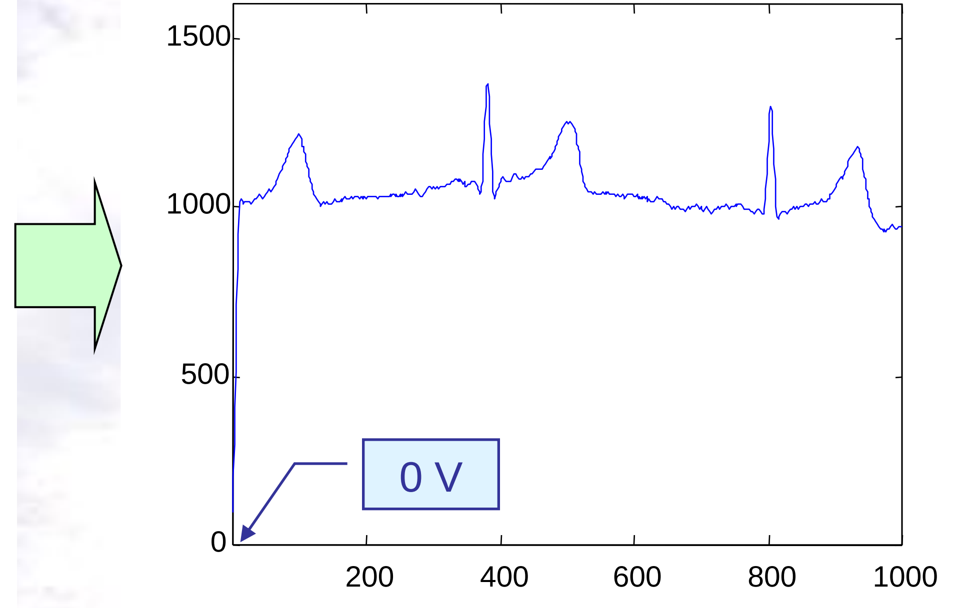
Frequency characteristics - examples



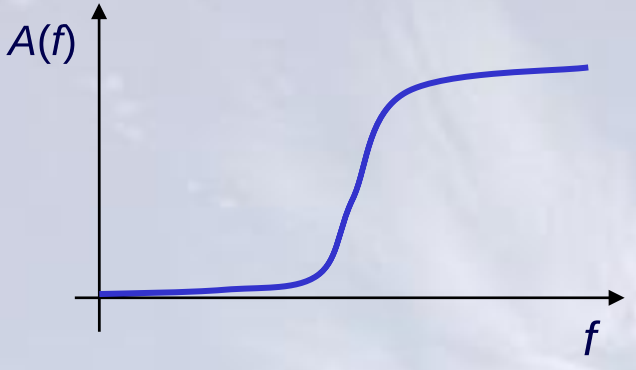
Low-pass filter

(*noise reduction*)



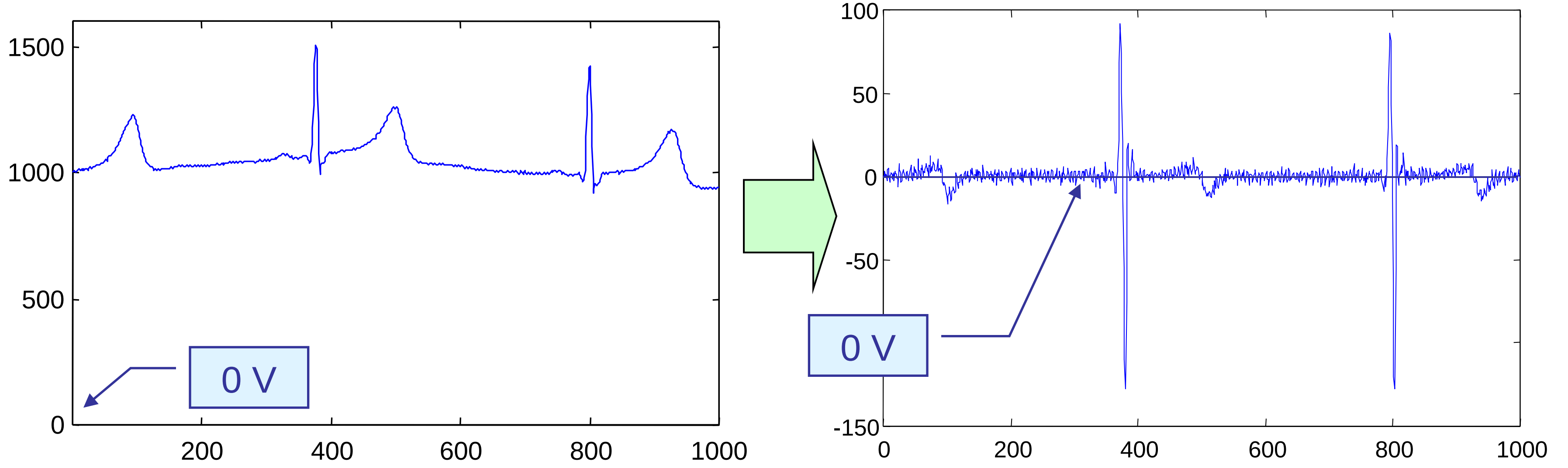


Frequency characteristics - examples



High-pass filter

(eg. *mean subtraction*)



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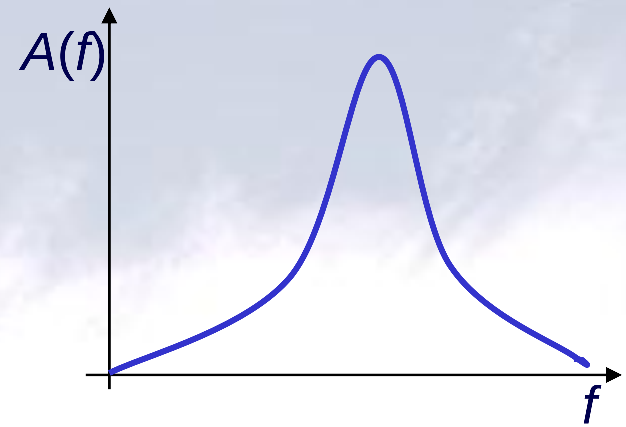
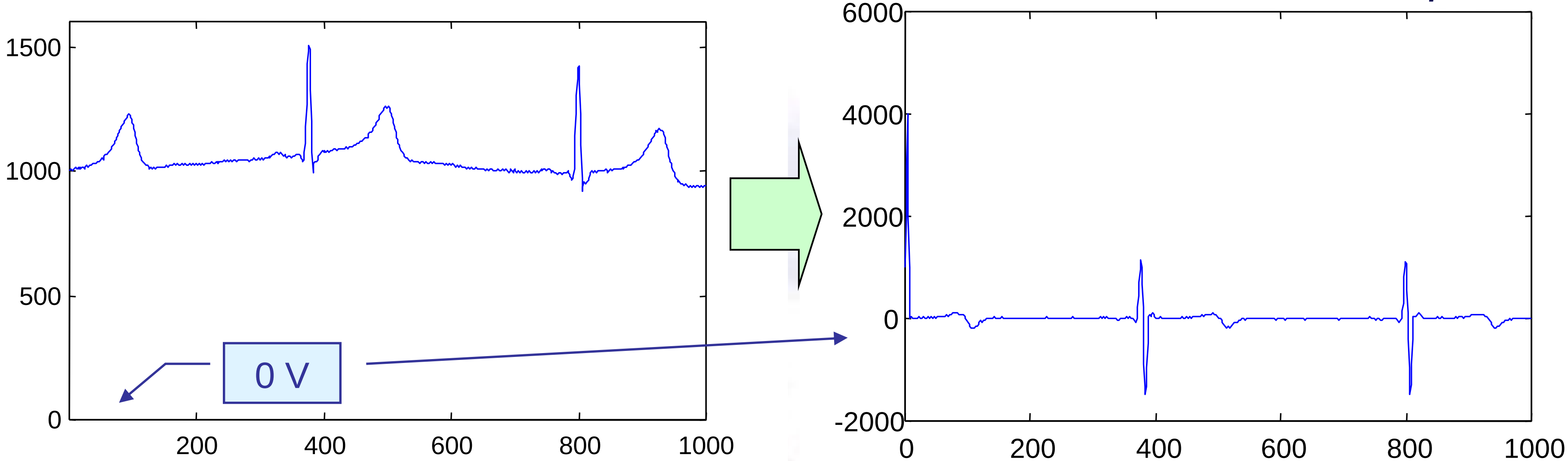
Frequency characteristics - examples

Narrow band-pass filter

(eg. *feature detection*)

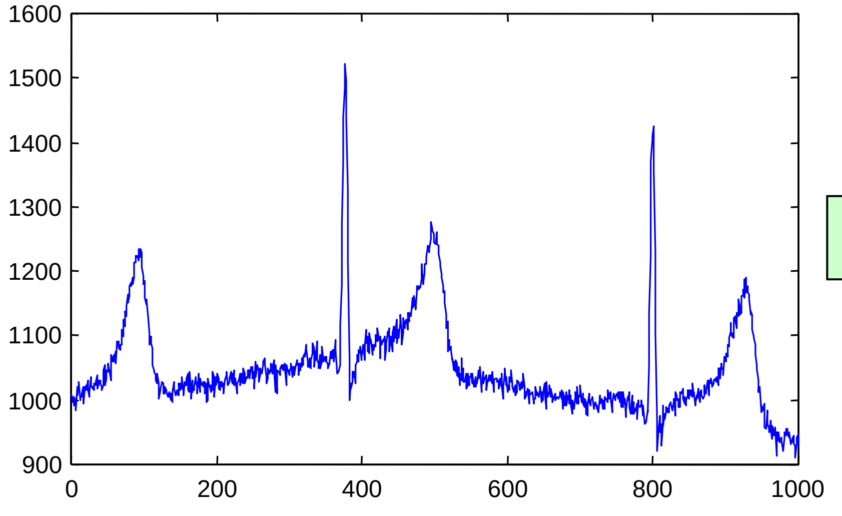


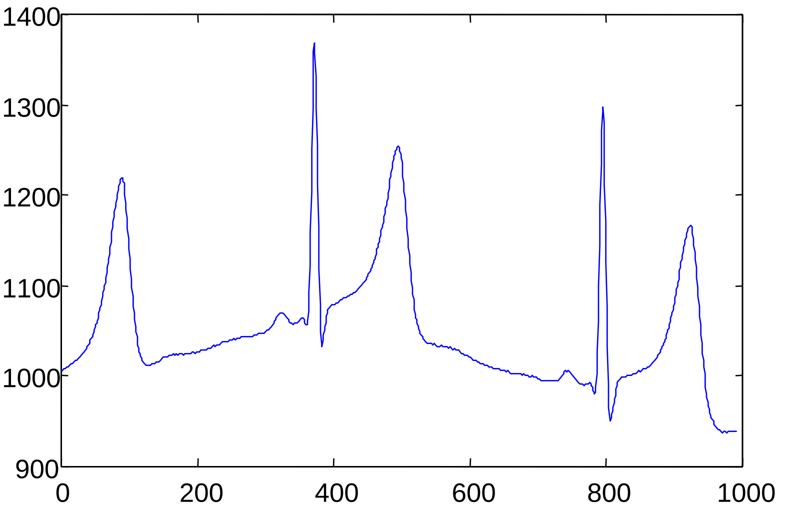


Frequency characteristics - examples

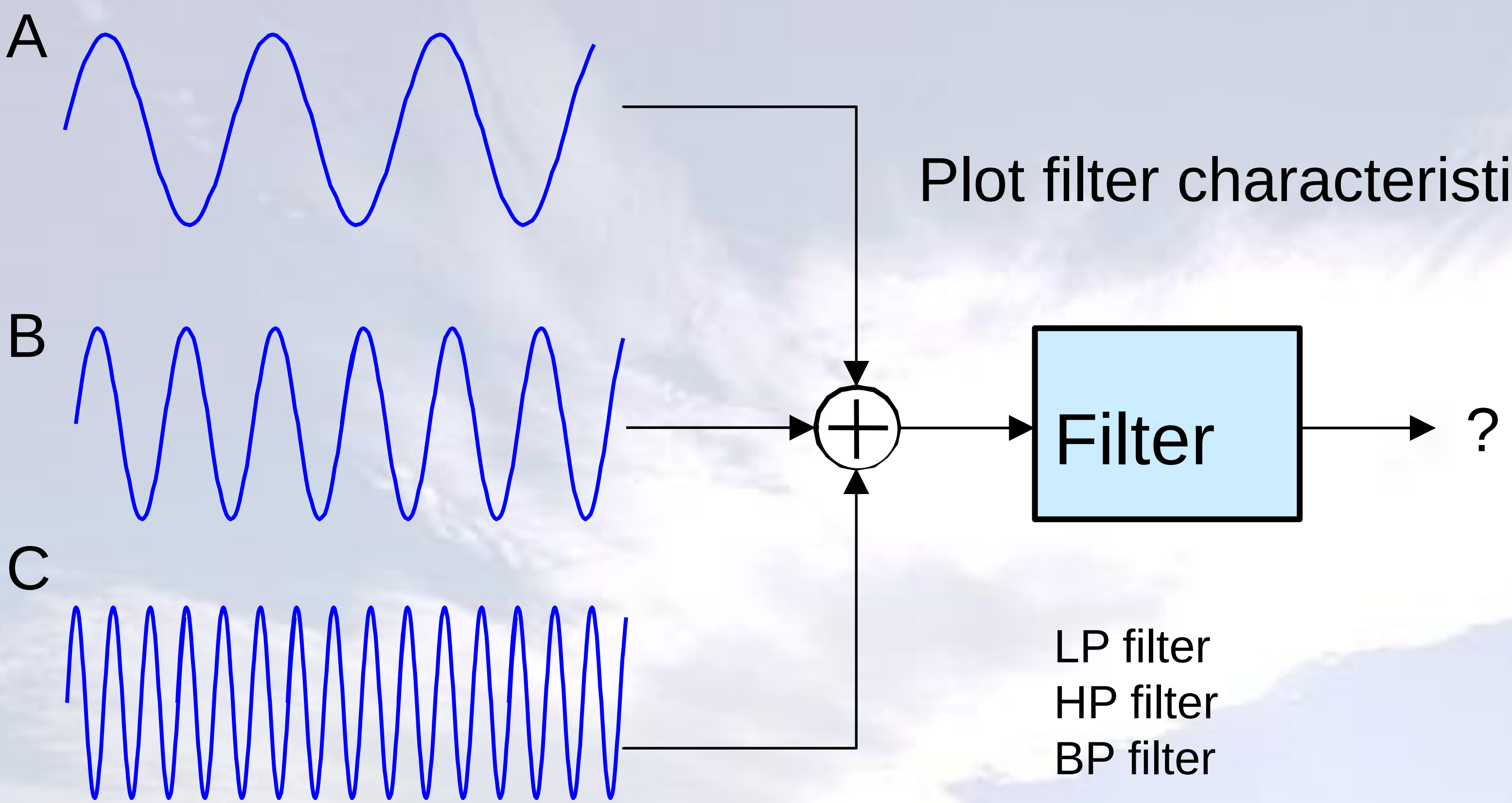
Band-stop filter

(eg. *Reduction of noise of precise frequency*)





Filter types



Application of filters to ECG processing

- **lowpass** (radio noise reduction, reduction of skeletal muscles activity noise)
- **highpass** (elimination of izoelectric line migration, f_g=0.5 Hz, see. ‘`ecg_mit.mat`’)
- **bandpass** (ECG feature detection, eg. P, T, QRS waves)
- **bandstop** (mains network interference reduction , f=50 Hz)

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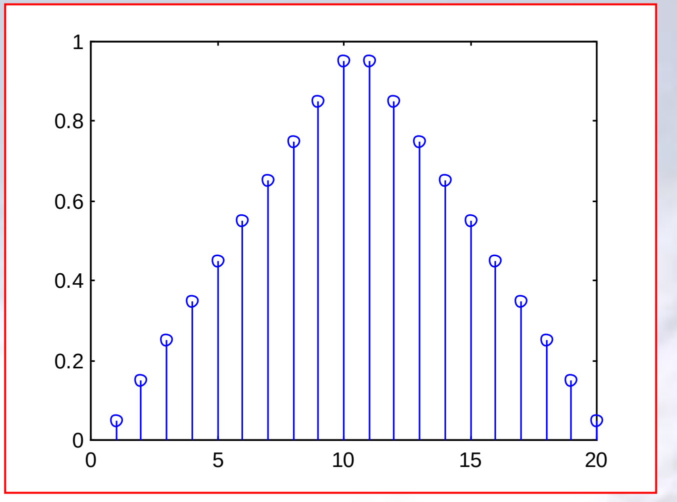
Digital filters – FIR i IIR

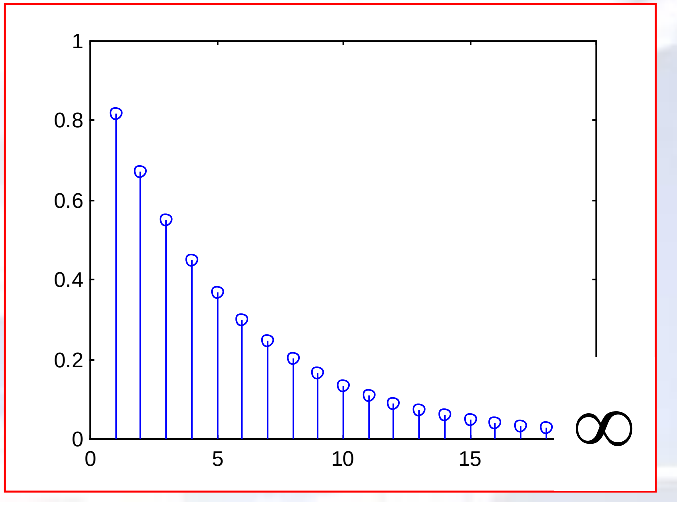
Another division:

Finite **I**mpulse **R**esponse (**FIR**)



Infinite **I**mpulse **R**esponse (**IIR**)

*co called **recursive filters***



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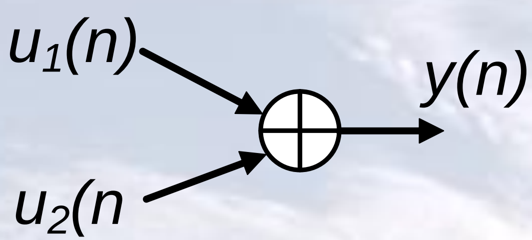
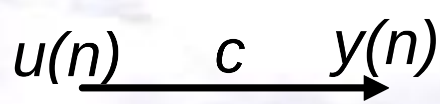
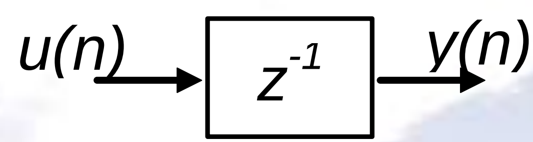
Digital filters – FIR i IIR

The following elementary functional operators are used in the design of a digital filter :

Summation	Multiplication by a constant	Unit delay





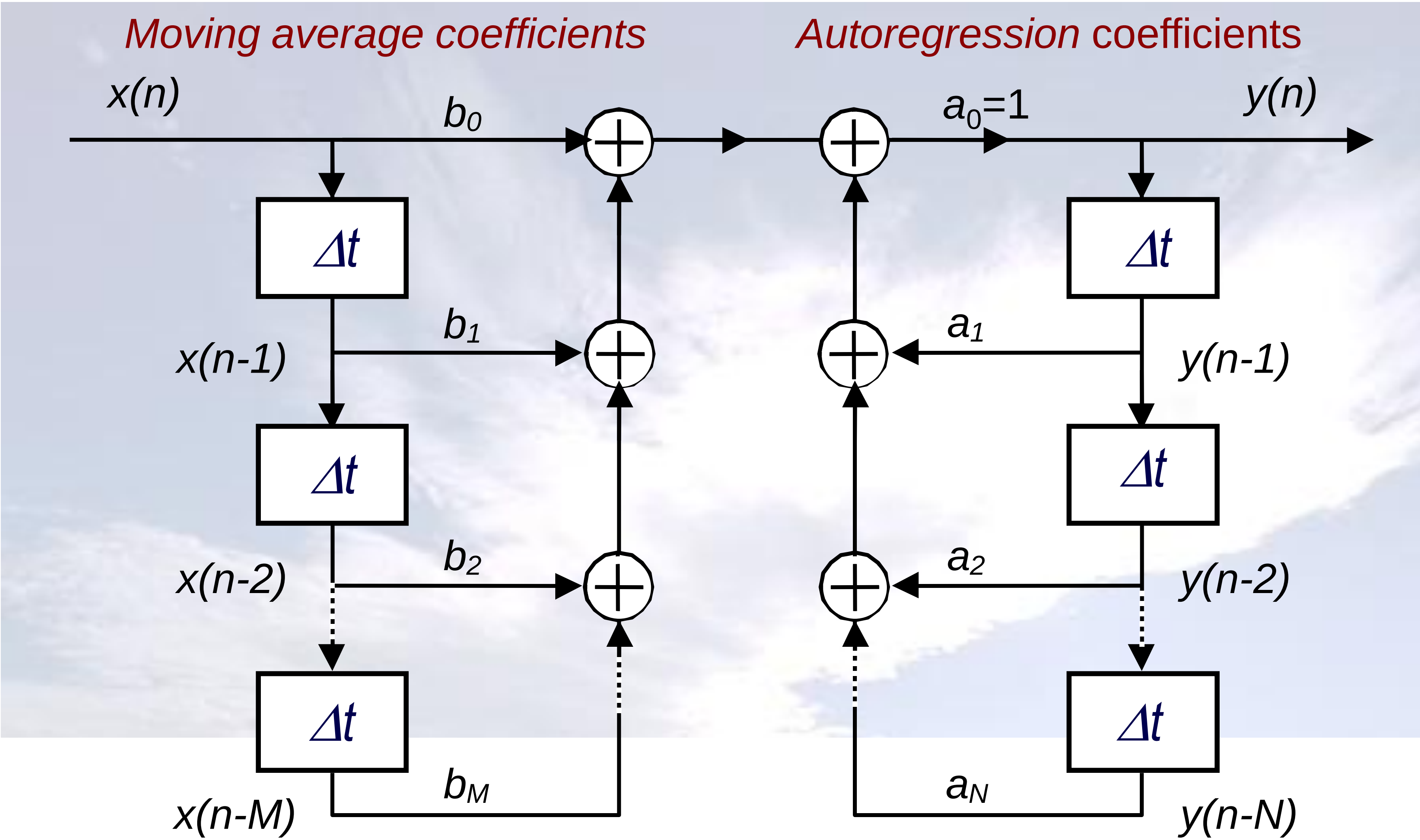


$$y(n)=u_1(n)+u_2(n)$$

$$y(n)=cu(n)$$

$$y(n)=u(n-1)$$

The idea of digital filtering



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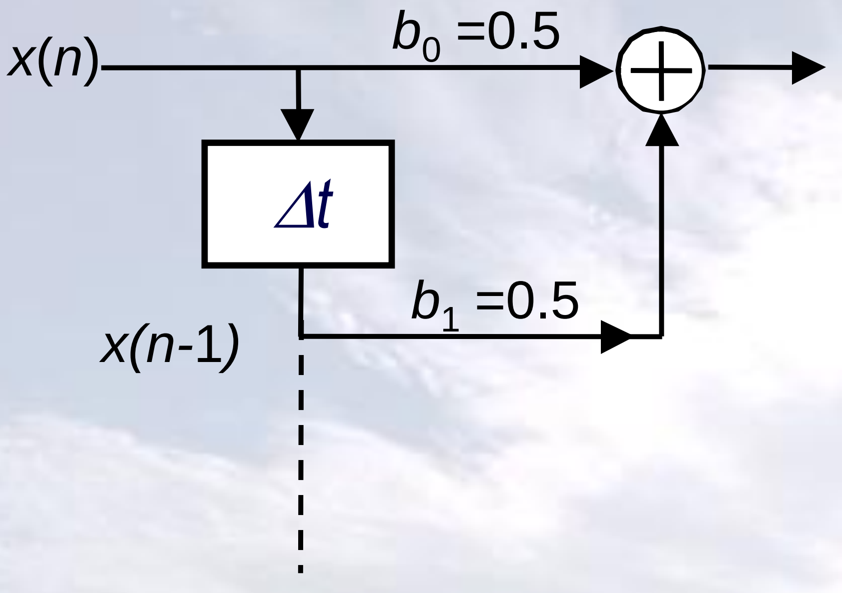
Finite impulse response (FIR) filter - example



LP:

$$y(n) = 0.5x(n) + 0.5x(n-1)$$

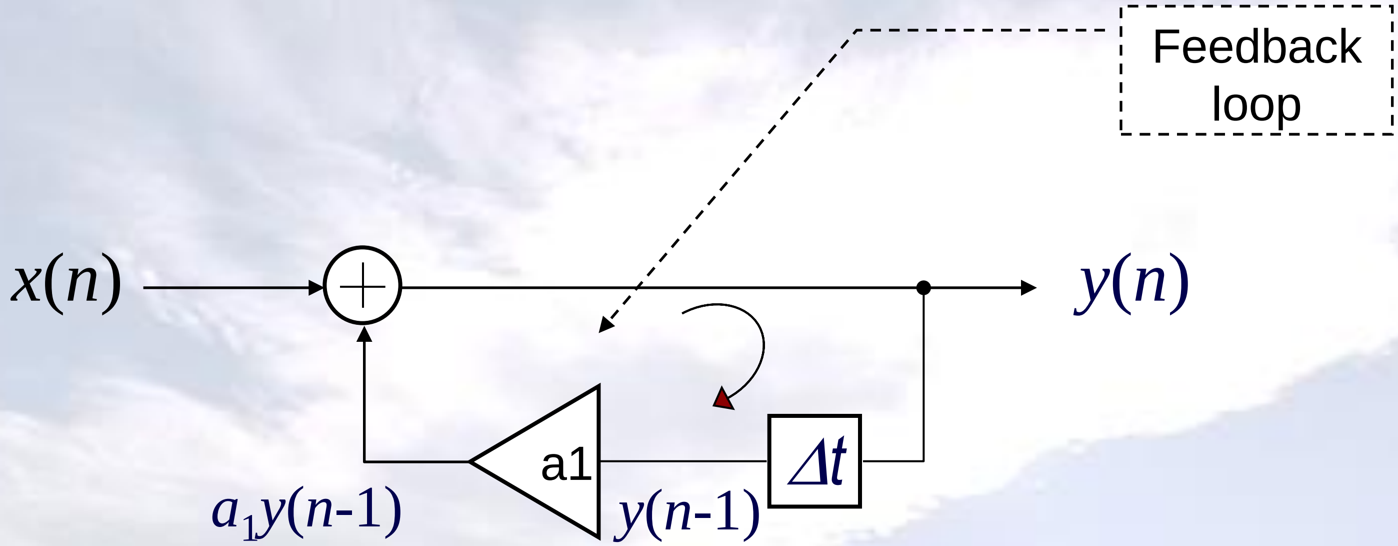
HP:

$$y(n) = 0.5x(n) - 0.5x(n-1)$$

BP: ?

Infinite response filter (IIR) - example

$$y(n) = a_1 y(n-1) + x(n)$$



Difference equation of a digital filter

$$\sum_{k=0}^{N} a(k)y(n-k) = \sum_{k=0}^{M} b(k)x(n-k)$$

which is equivalent to a difference equation:

$a[0]=1$

$$a(0)y(n) = \sum_{k=0}^{M} b(k)x(n-k) - \sum_{k=1}^{N} a(k)y(n-k)$$

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Difference equation of a digital filter

$$y(n)=\sum_{k=0}^{M} b(k)x(n-k)-\sum_{k=1}^{N} a(k)y(n-k)$$

If all $a(k)$ coefficients are equal zero, then the difference equation defines a FIR filter and an IIR filter otherwise.

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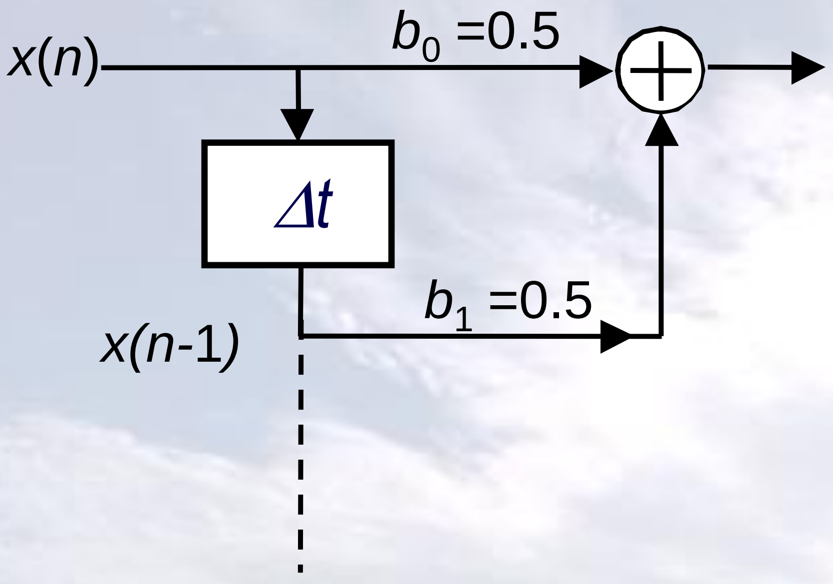
Finite response filter (FIR) - example



LP:

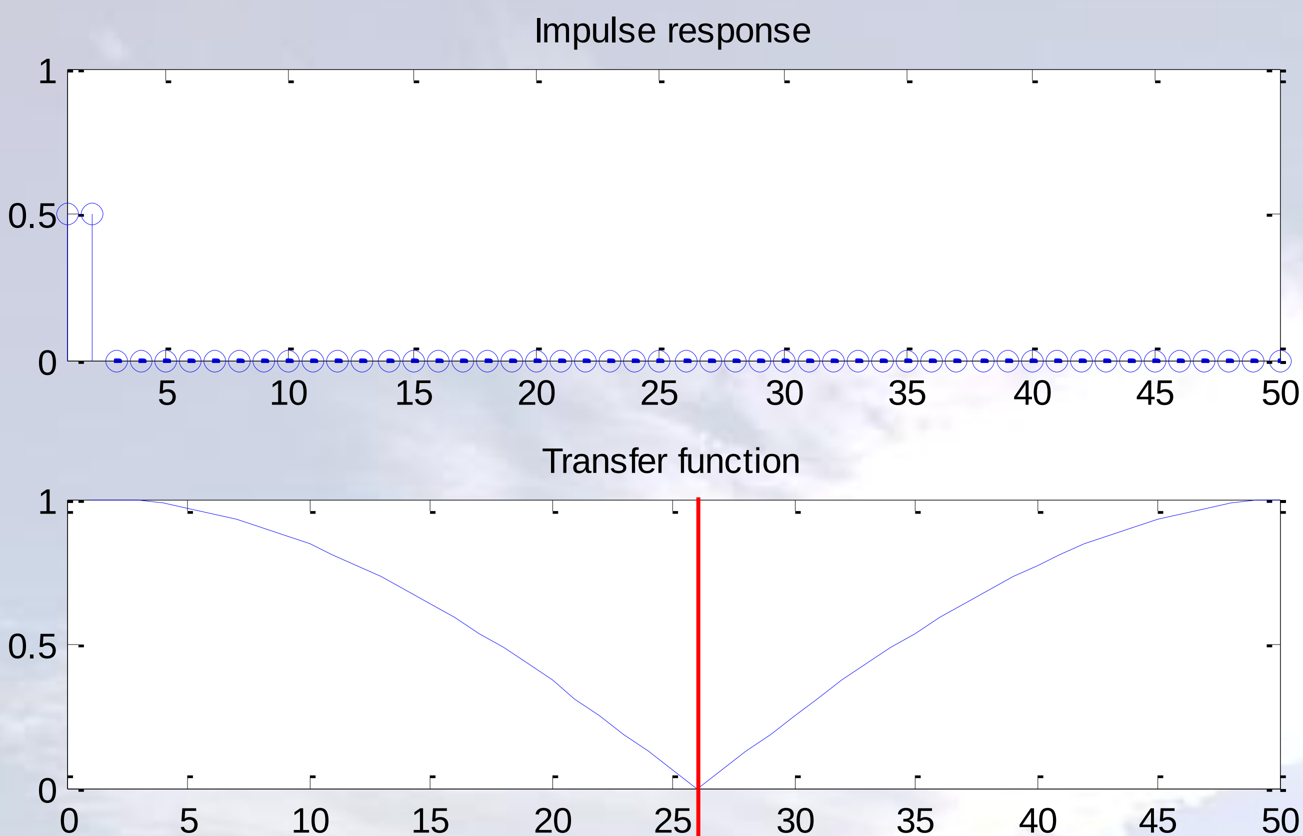
$$y(n) = \underbrace{0.5}_{b_0} x(n) + \underbrace{0.5}_{b_1} x(n-1)$$

HP:

$$y(n) = \underbrace{0.5}_{b_0} x(n) \underbrace{-0.5}_{b_1} x(n-1)$$

BP: ?

Finite response filter (FIR) – low pass filter



$$h_L = [0.5 \quad 0.5]$$

$$H_L = FFT(h_L)$$

$$\boxed{N/2+1 \rightarrow f_s/2}$$

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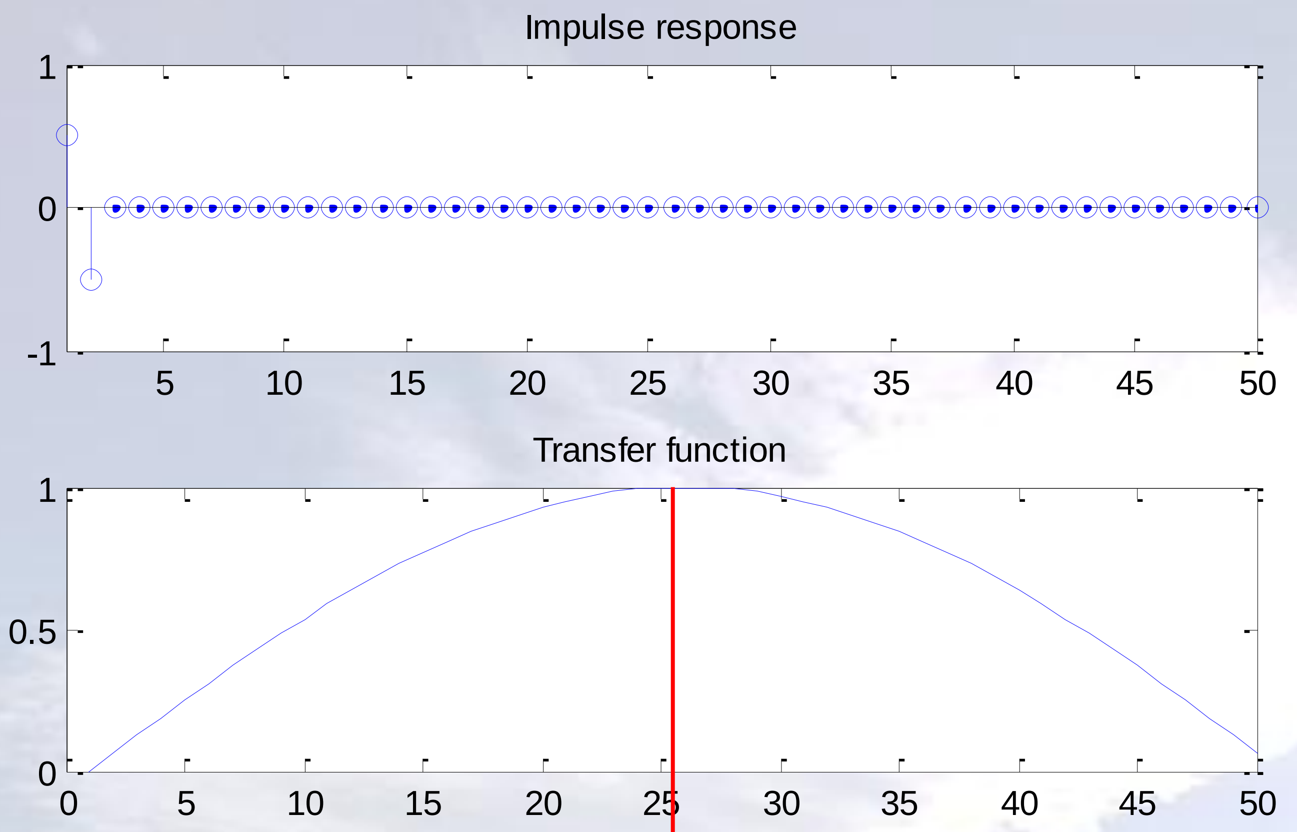
Finite response filter (FIR) – high pass filter



$$h_H = [0.5 \quad -0.5]$$

$$H_H = FFT(h_H)$$

$$\boxed{N/2+1 \rightarrow f_s/2}$$

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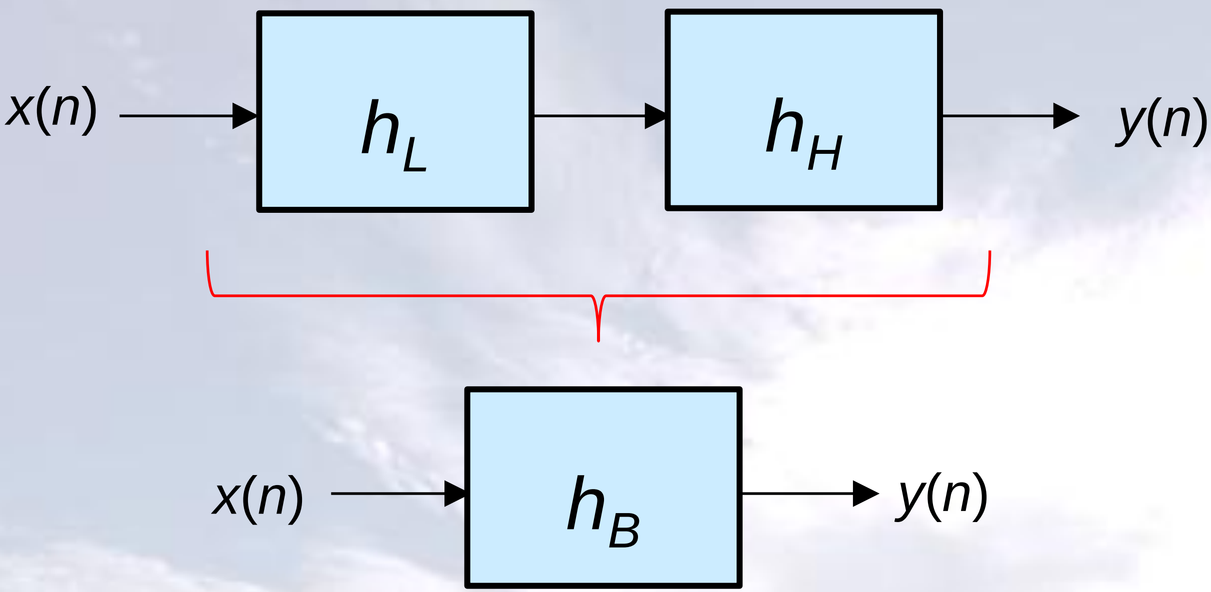
Filters in a cascade connection



$$h_B = h_L * h_H$$

$$h_B = h_L * h_H \quad \rightarrow \quad H_B(\omega) = H_L(\omega) \cdot H_H(\omega)$$

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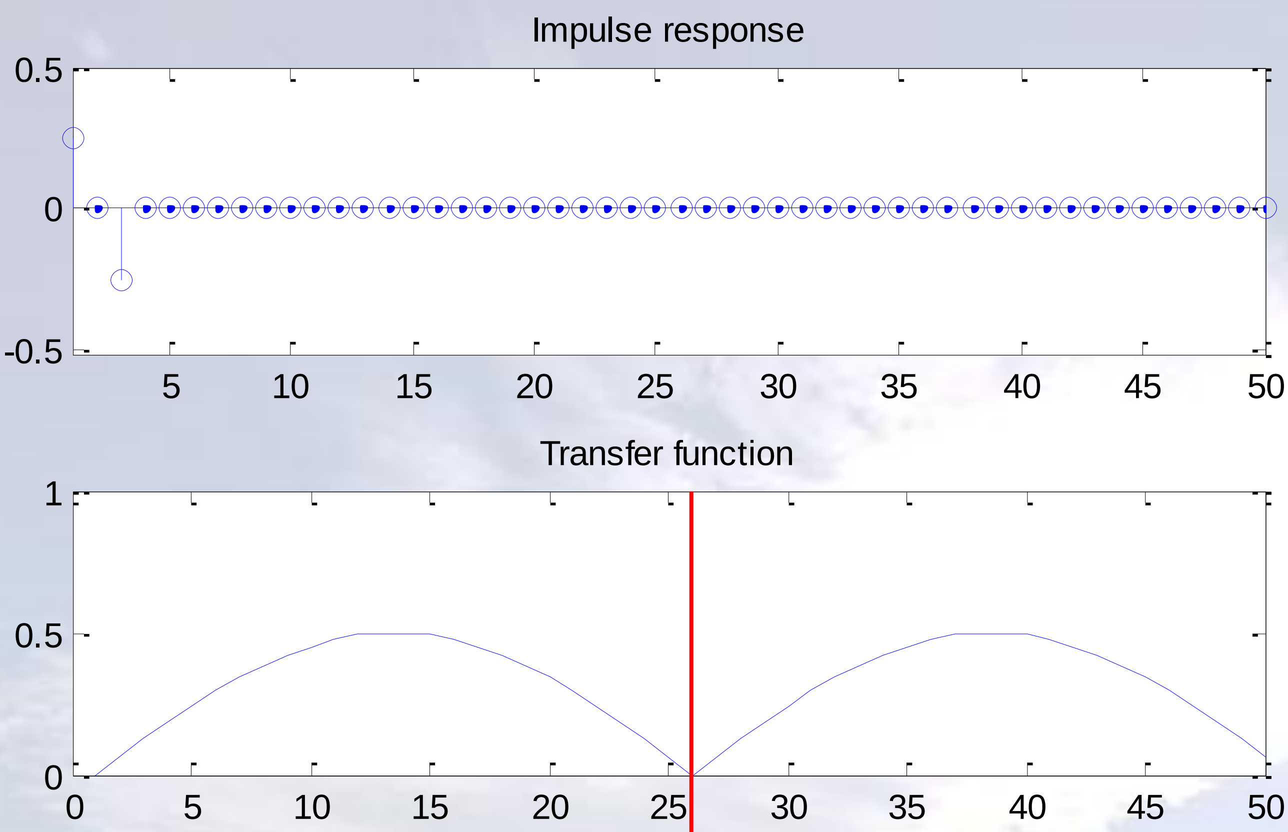
Finite response filter (FIR) – band pass filter



$$h_B = [0.25 \quad 0 \quad -0.25]$$

$$H_B = FFT(h_B)$$

$$\boxed{N/2+1 \rightarrow f_s/2}$$

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Simple FIR filter example

Moving average filter equation is given:

$$y(n)=\frac{1}{5}\left[x(n-4)+x(n-3)+x(n-2)+x(n-1)+x(n)\right]$$

Determine the vectors of coefficients *a* and *b* for this filter .

Solution (simple low pass FIR filter):

a=[1];

b=[0.2 0.2 0.2 0.2 0.2];

Simple FIR filter example

What is the impulse response of this filter:

$$y(n) = \frac{1}{5}\left[x(n-4) + x(n-3) + x(n-2) + x(n-1) + x(n)\right]$$

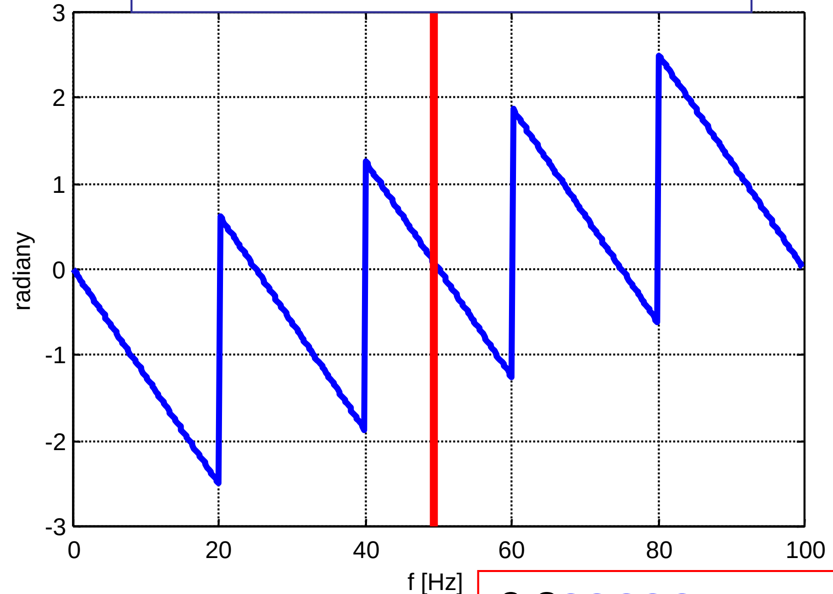
Answer:

h=[0.2 0.2 0.2 0.2 0.2];

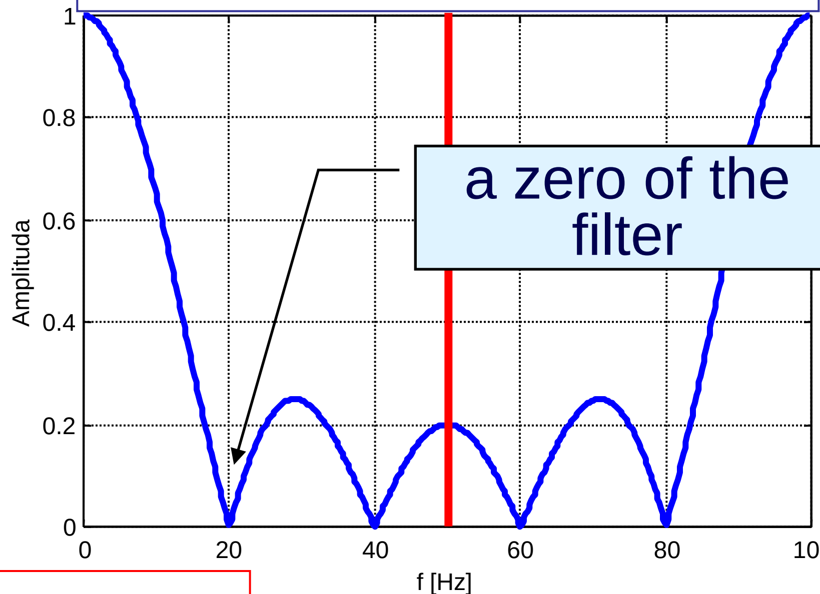
Important conclusion: For FIR filters the filter coefficients are equal to filter's impulse response

Simple FIR filter example

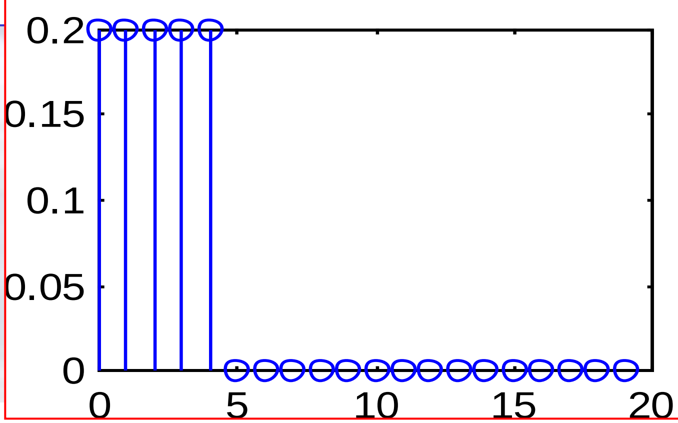
Phase characteristic



Amplitude characteristic



Impuse response of the filter



Simple FIR filter example

Exercise (cont.):

Observe the amplitudes and phase shifts of the output characteristics of the filter for the following inputs:

- sinusoid with f=1 Hz
- sinusoid with f=5 Hz
- sinusoid with f=20 Hz

Confirm that filtering (convolution) in time domain is equivalent to multiplication of spectrum of the impulse response of the filter with the spectrum of the signal.

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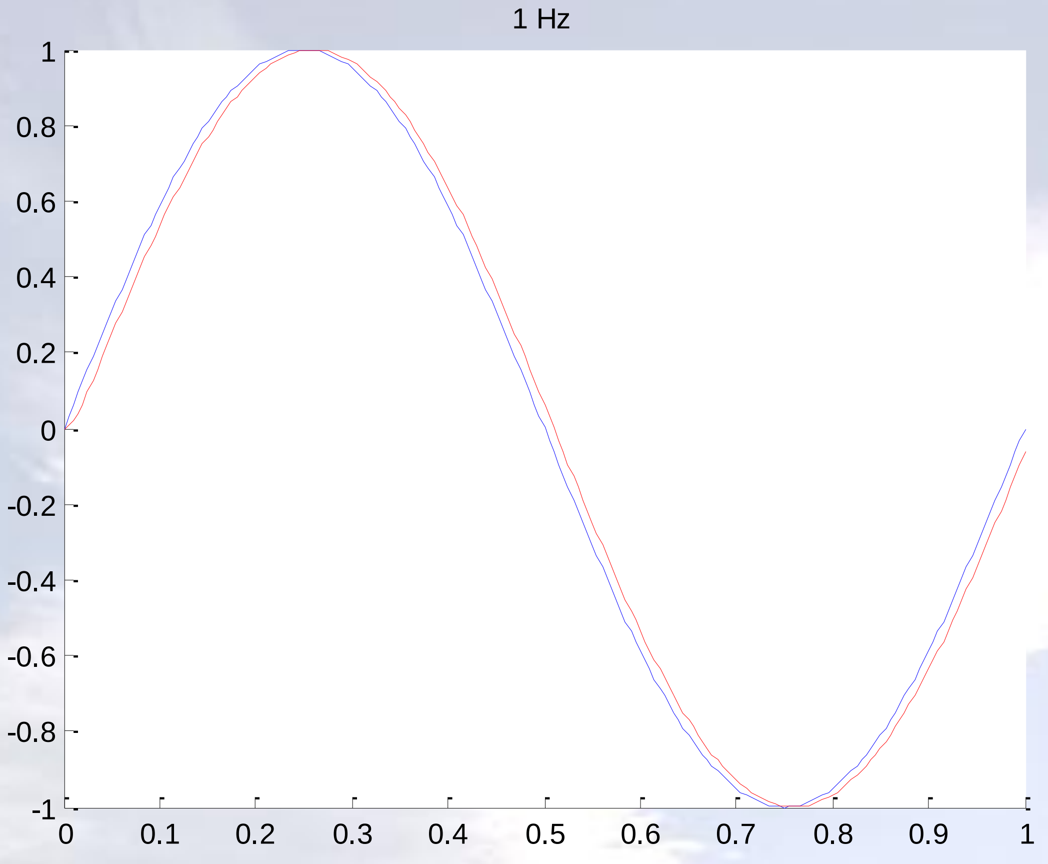
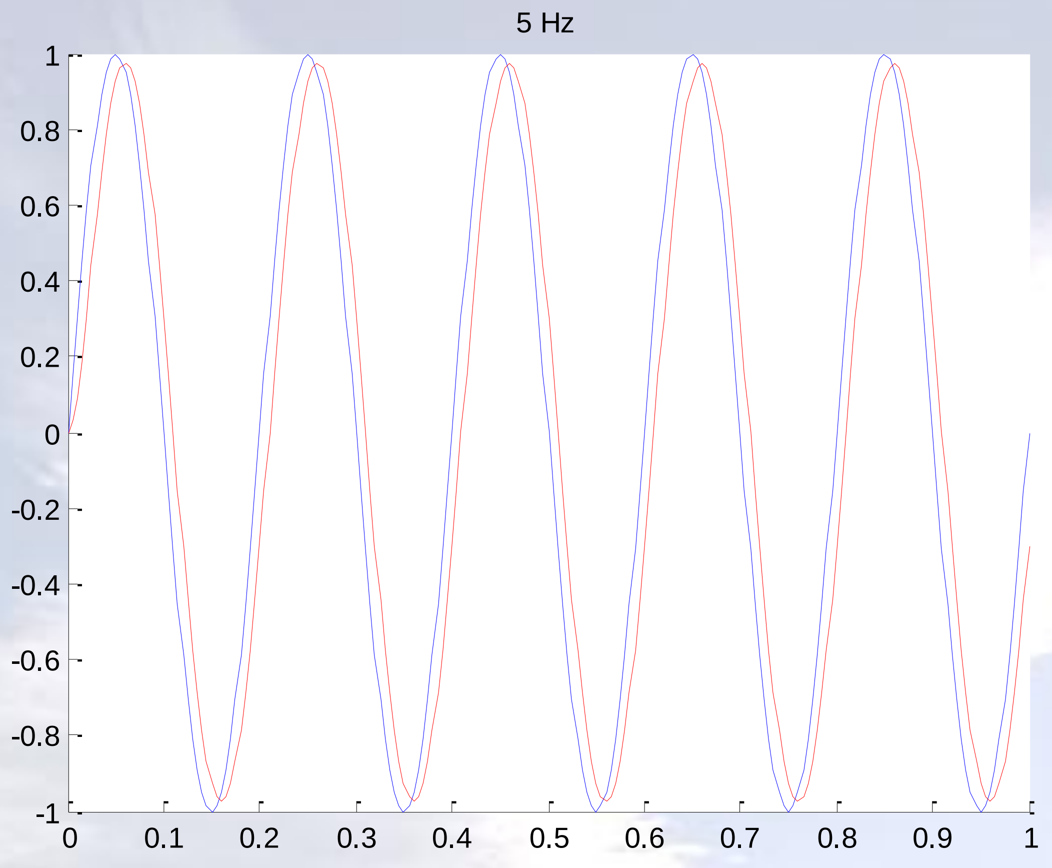
Simple FIR filter example



Simple FIR filter example



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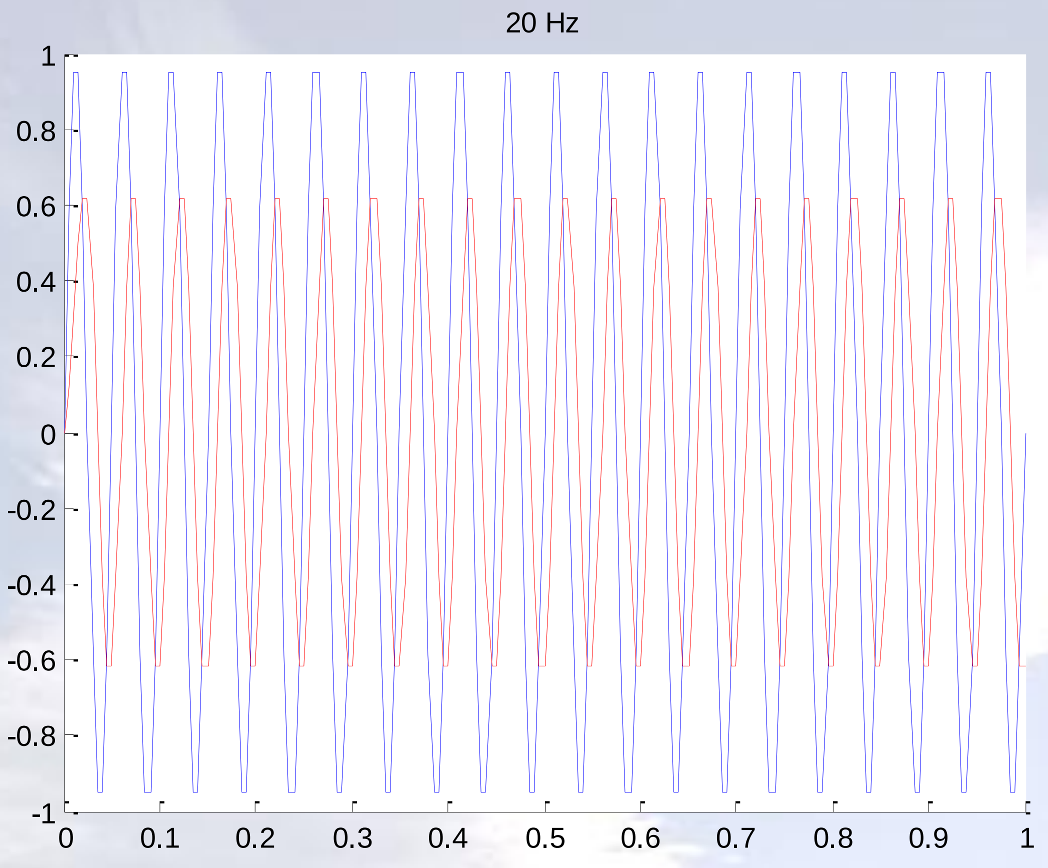
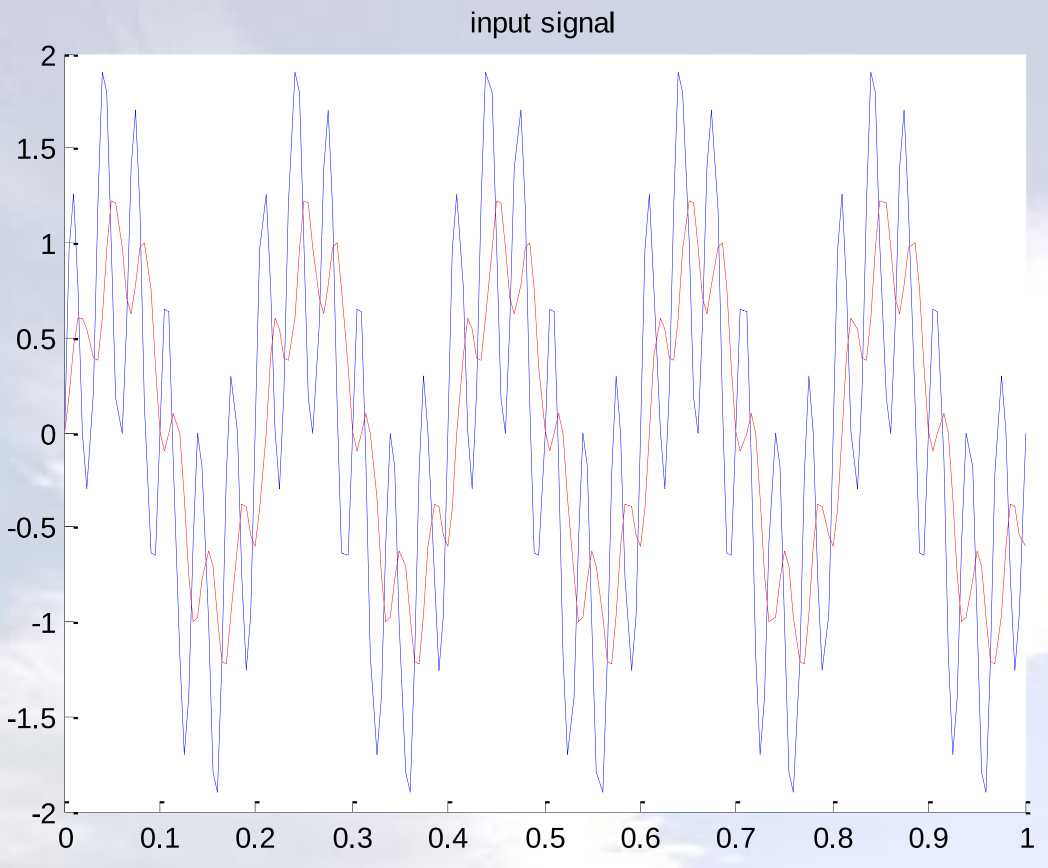
Simple FIR filter example



Simple FIR filter example



Linear phase condition for FIR filters

FIR filters may be designed so that their phase characteristic is linear. For this purpose the ‘symmetry’ condition for the filter coefficients is to be fulfilled.

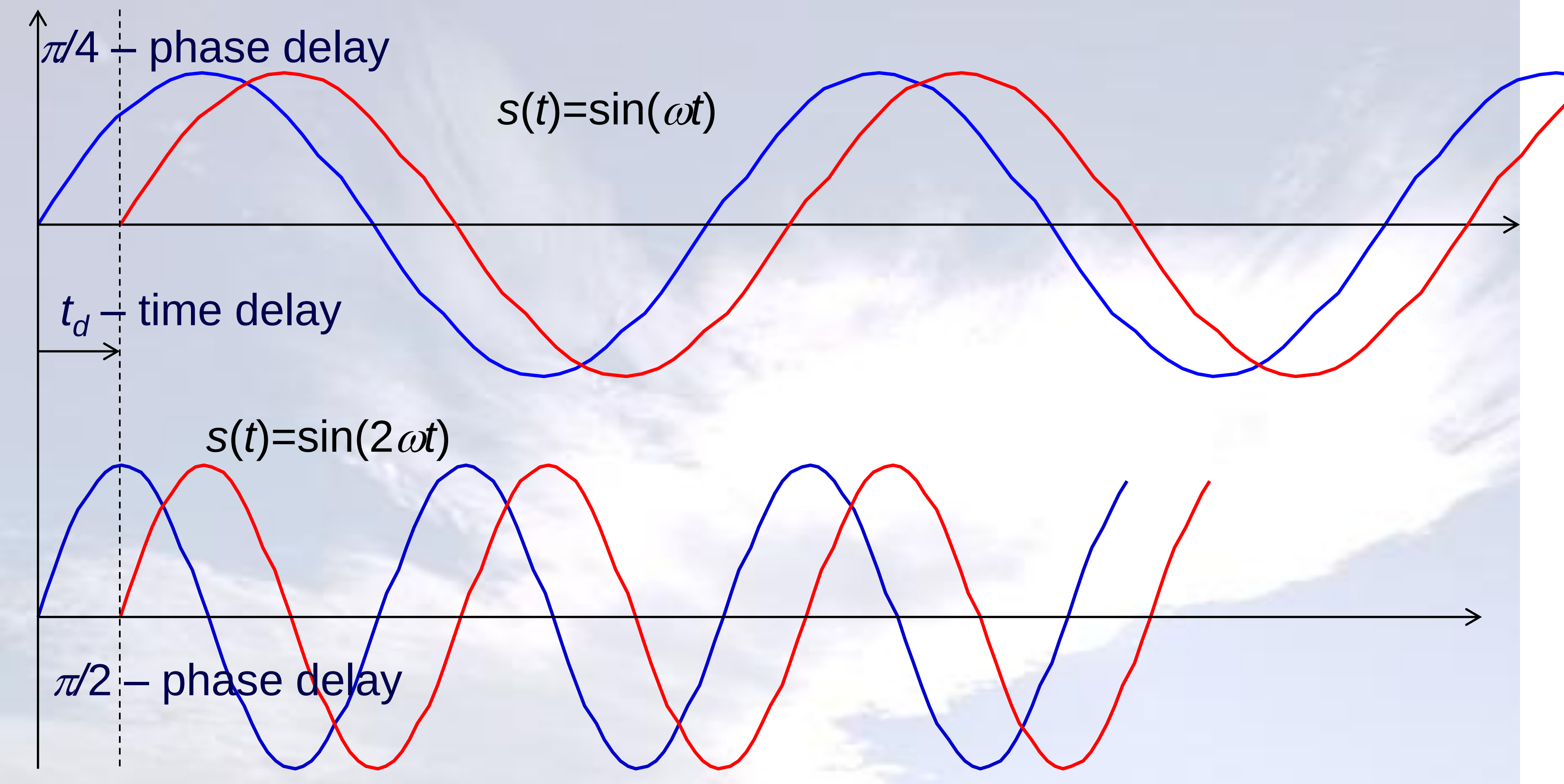
$$\boxed{h(M-k)=h(k)} \quad \text{for} \quad k=0,1,\ldots,M$$

$$\boxed{h(M-k)=-h(k)} \quad \text{for odd or even } M$$

What is the advantage of the linear filter phase characteristic?

! <u>The delay of the signal on the output is independent on its frequency.</u>

Linear phase → constant delay of all frequencies



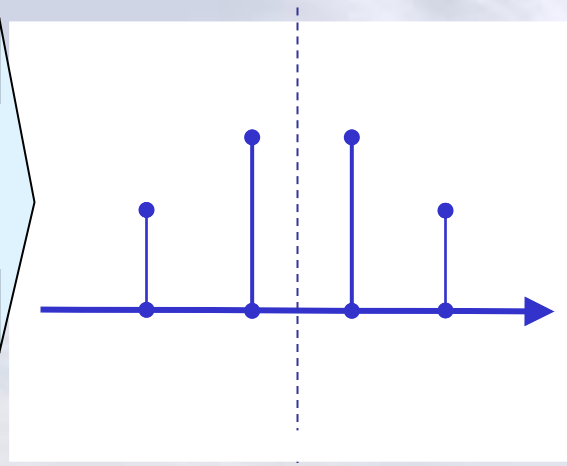
For linear phase filters all frequencies are delayed by the same time interval!

Linear phase condition for FIR filters

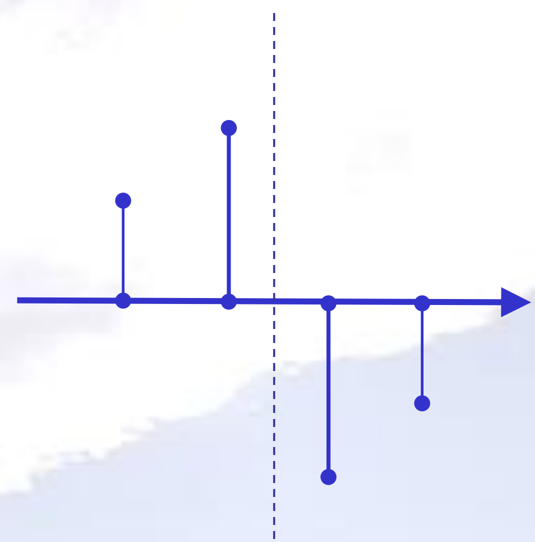
$$h(M-k)=h(k)$$

$$h(M-k)=-h(k)$$

for $k=0,1,\dots,M$



discrete time



discrete time

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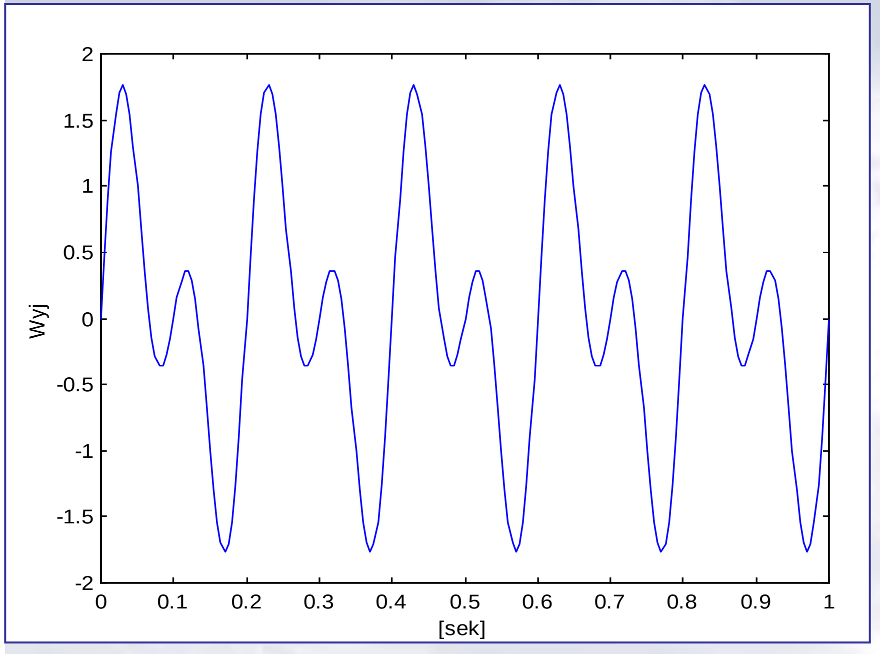
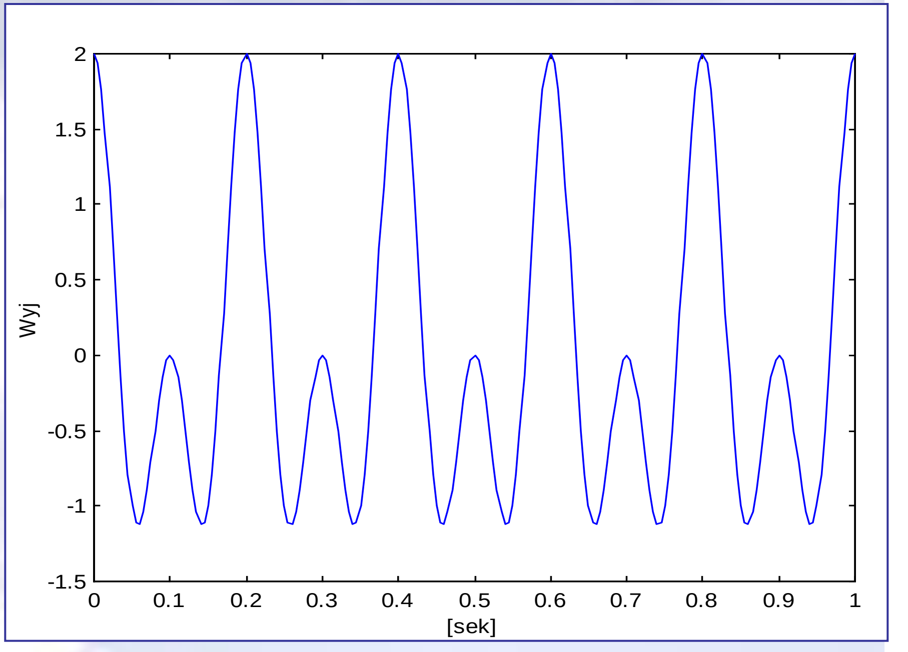
Phase distortion

$$x_{we} = sin(10\pi t) + sin(20\pi t)$$

$$x_{wy} = sin\left(10\pi t + \frac{\pi}{2}\right) + sin\left(20\pi t + \frac{\pi}{2}\right)$$





Nonlinear phase distortion

(eg. significant distortion of the acoustic signal)

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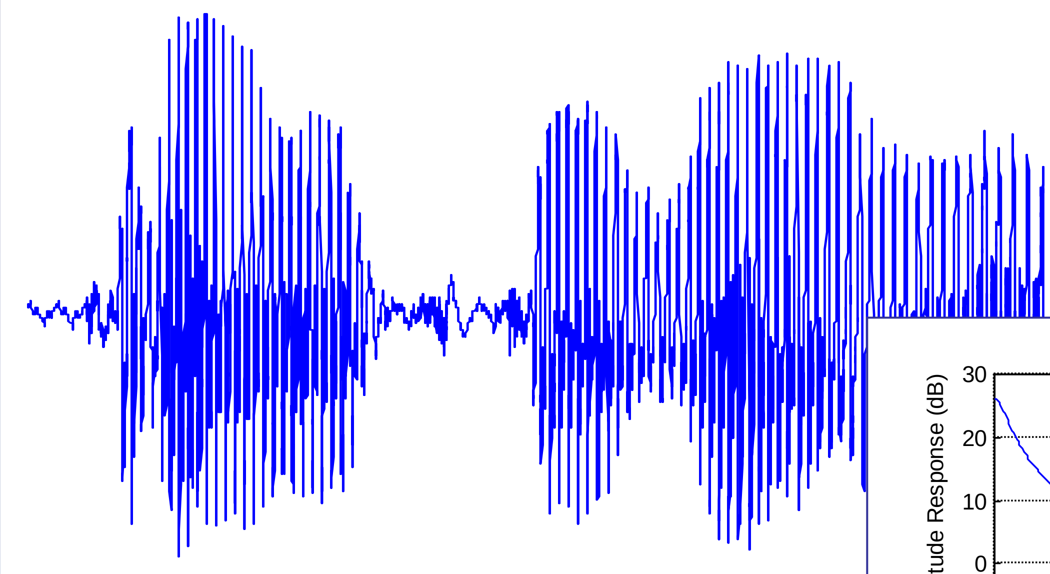
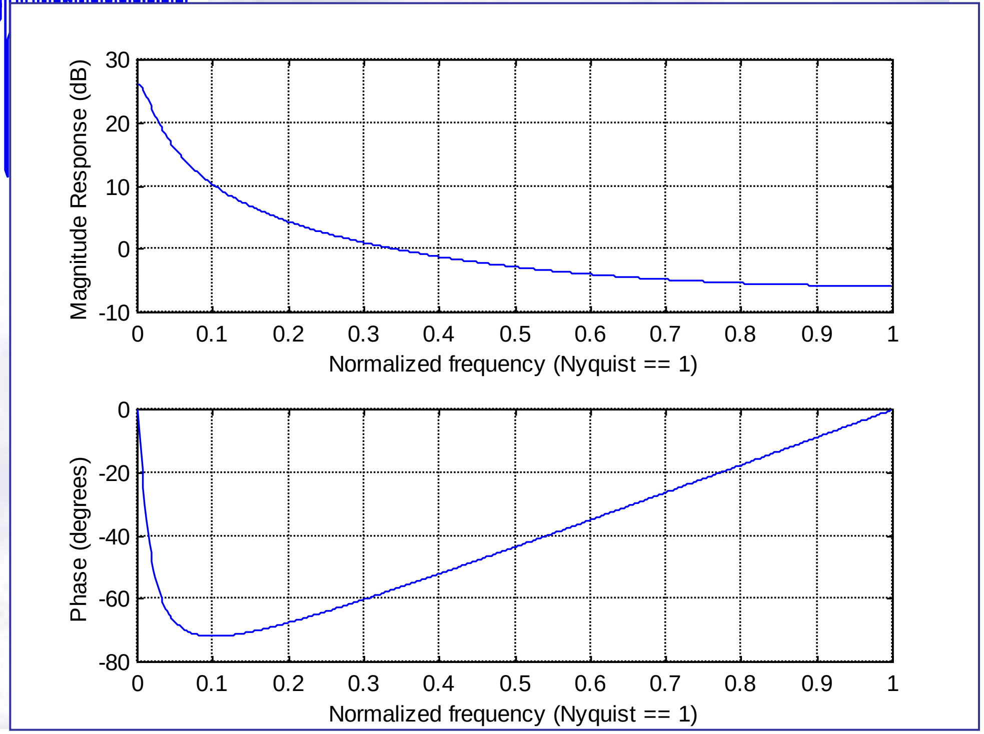
Phase distortion - example





Other FIR filter examples

Exercise cont.:

Determine the characteristics of the filters of the following impulse response:

- h=0.25*[1 2 1] , so called von Hann filter (or Gaussian filter)
- h=[1 -2 1] , ~ second derivative of the signal

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FIR filter design

from scipy.signal import firwin

Where *f* is a normalised frequency *f*=<0,1>, where 1 corresponds to *fs*/2

Examples

Low-pass from 0 to f:

```
>>> firwin(numtaps, f)
```

Use a specific window function:

```
>>> firwin(numtaps, f, window='nuttall')
```

High-pass ('stop' from 0 to f):

```
>>> firwin(numtaps, f, pass_zero=False)
```

Band-pass:

```
>>> firwin(numtaps, [f1, f2], pass_zero=False)
```

Band-stop:

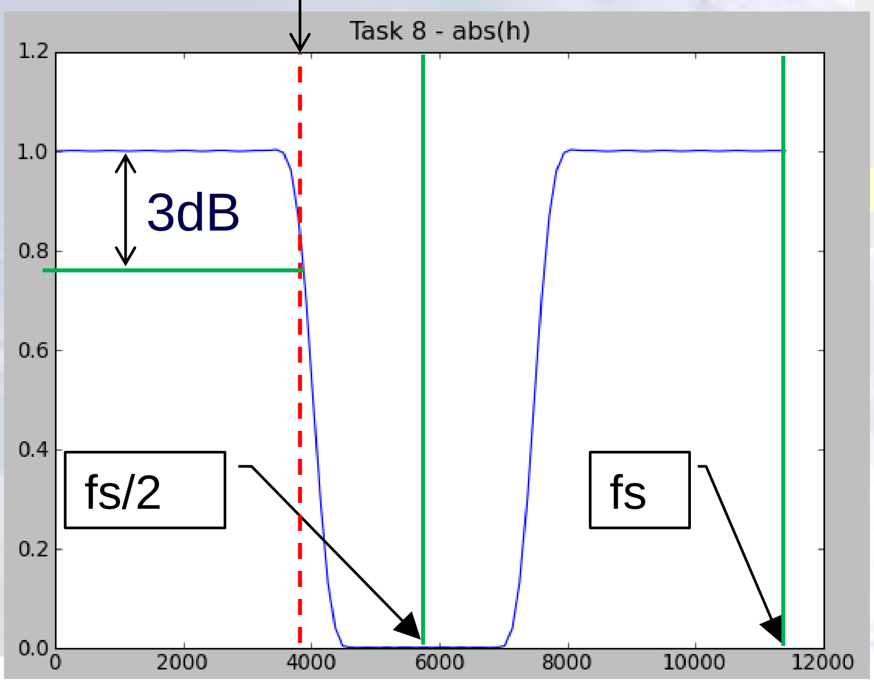
```
>>> firwin(numtaps, [f1, f2])
```

FIR filter design - example

M=40
– no. of taps (filter order)

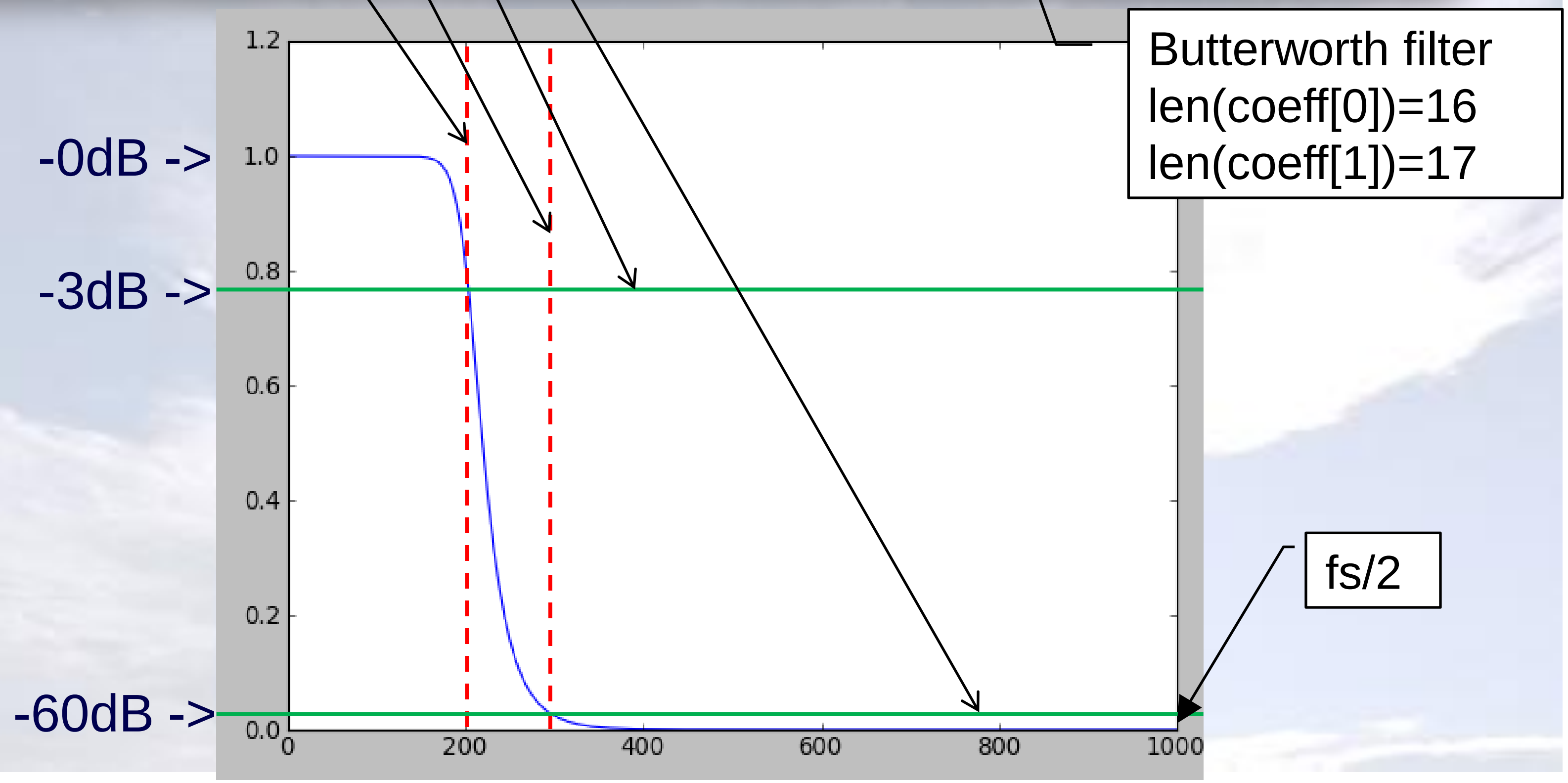
f_{off}=0.7*fs/2

```python
from pylab import figure,plot, show
from numpy import arange
from scipy.signal import firwin, freqz
b=firwin(40,0.7)
N=100
fs=1140
w,h=freqz(b,1,N, whole='True')
f=arange(0,fs,1.0*fs/N)
figure()
plot(f,abs(h))
show()
```



IIR filter design - example

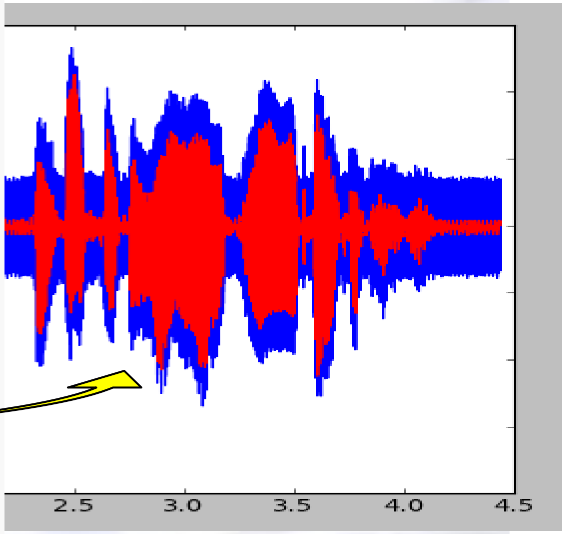
```
from scipy.signal import firwin, iirdesign
coeff=iirdesign(0.2,0.3,3,60,analog=0,ftype='butter',output='ba')
w,h=freqz(coeff[0],coeff[1])
plot(abs(h))
```



Filter implementation

```
from scipy.io.wavfile import read as read_wav
from numpy import array, arange
from pylab import show, plot, title, figure
from scipy.signal import lfilter, freqz, convolve, firwin

fs, audio = read_wav('noise_voice.wav')

fs=11400
N=50580
T=float(N)/fs #time period definition
t=arange(0,T,1.0/fs)

figure(1)
plot(t,audio) #plot the noisy audio signal


b=firwin(40,0.7) #coefficients of the low-pass FIR filter

y = lfilter(b,[1],audio) # filtering of the audio signal


plot(t,y,'r')     #plot the filtered audio signal
show()
```

a[0]=1

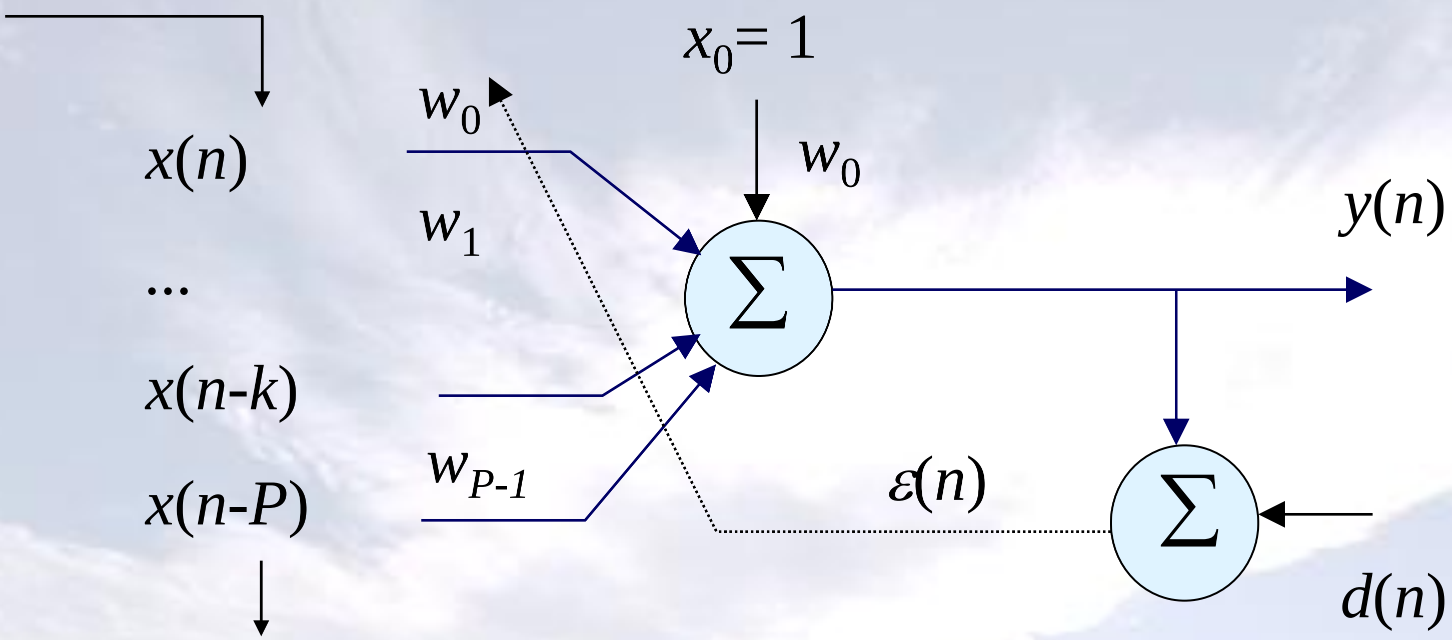


IIR and FIR filters comparison

FIR	IIR
Stable (from definition)	Can be unstable
Simple design	Complex design
condition for linear phase easy to satisfy	Nonlinear phase
Steep characteristic possible for very high orders of the filter	Very steep characteristic possible for low orders of the filter
Finite precision of filter coefficients is not a significant problem	Problems with implementation due to finite precision of filter coefficients.

Adaptative filters

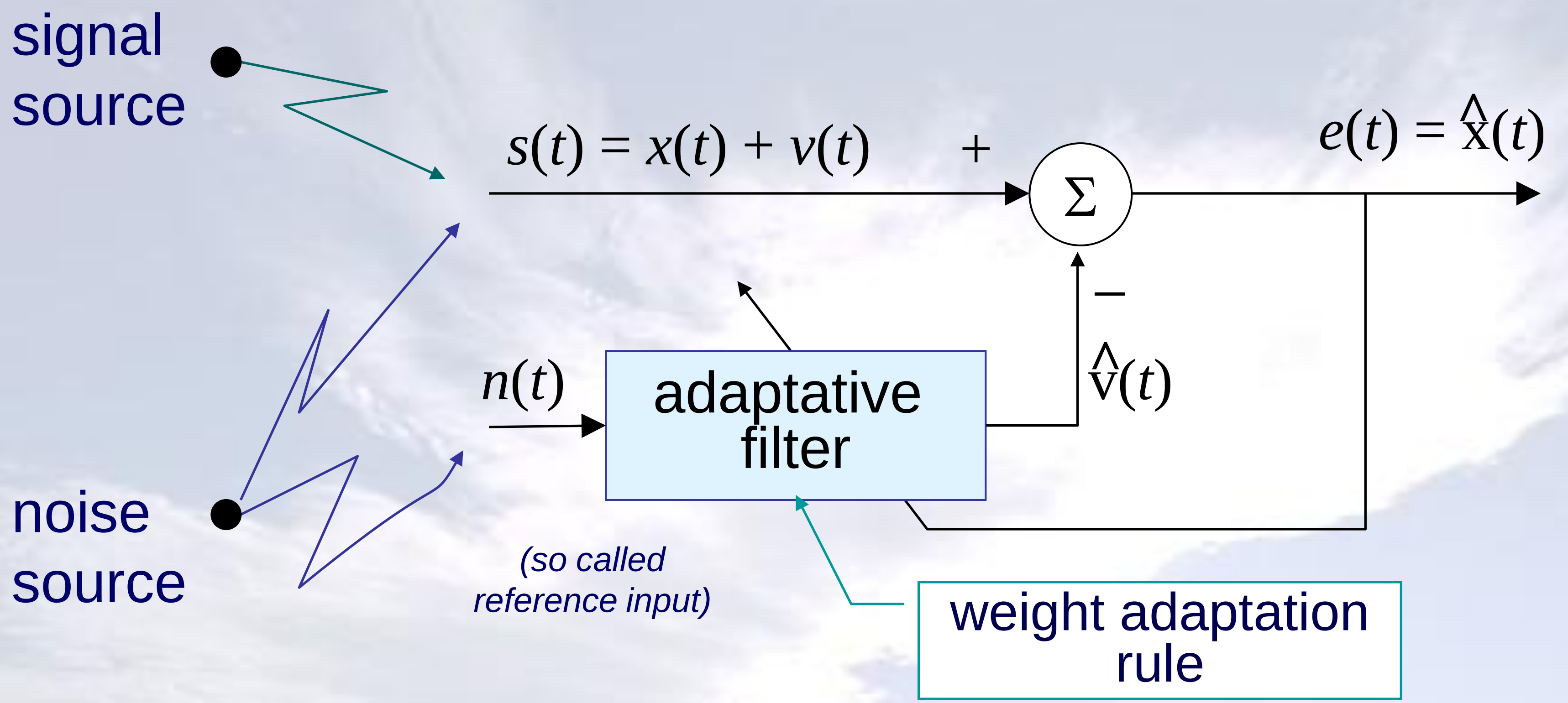
Successive signal samples



Target function:

$$E[d(n) - \mathbf{w}^T\mathbf{x}] = [(d(n) - y(n))^2] = E[\varepsilon^2] \rightarrow min$$

Adaptative noise reduction



$$\Delta w(t) = -\eta \nabla e(t)$$

Adaptative noise reduction

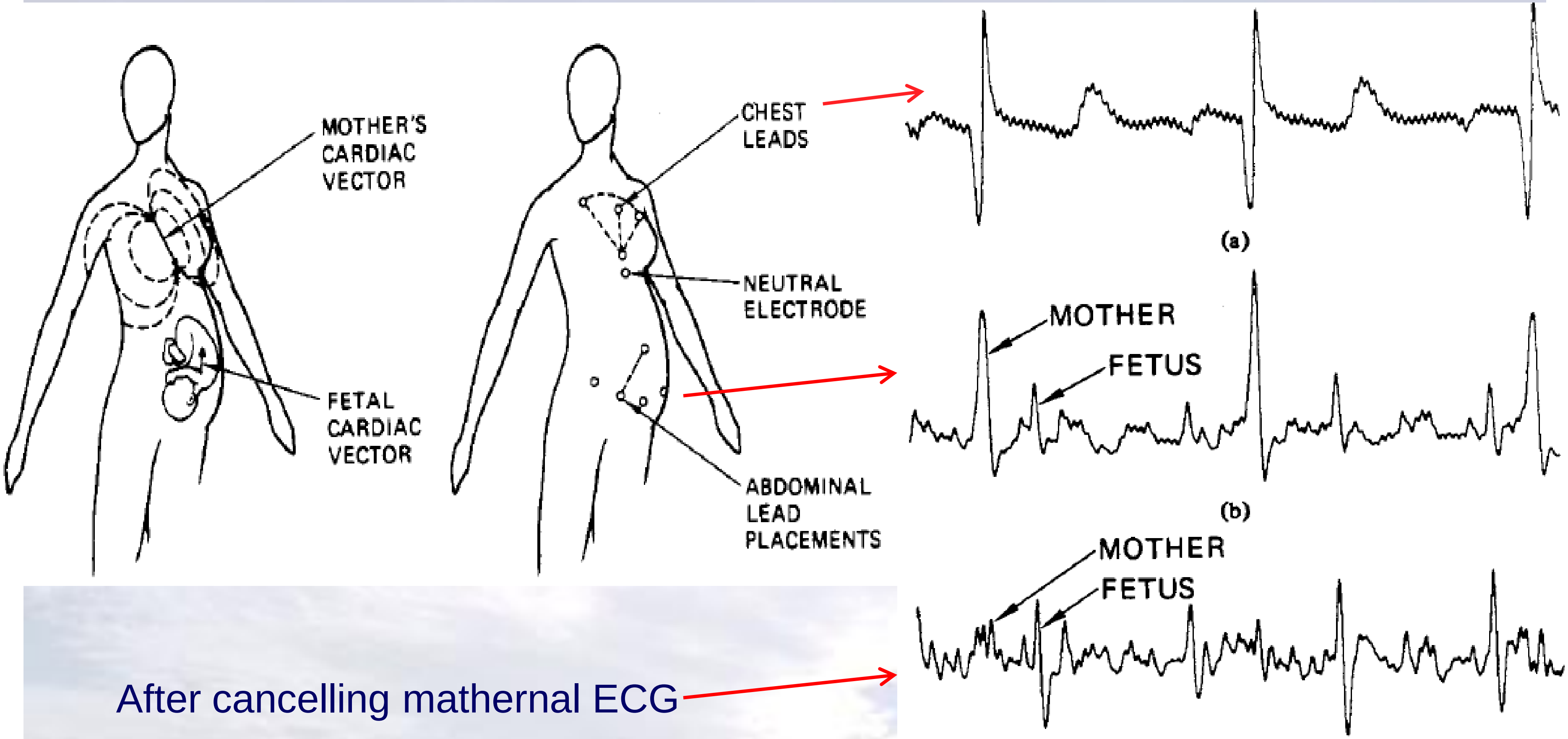
Minimal $e(t)$ corresponds to the most effective (according to the minimum mean square error) noise reduction:

$$e(t) = s(t) - \hat{v}(t) = x(t) + v(t) - \hat{v}(t)$$

$$E[e^2] = E[x^2] + \underbrace{2E[x(v-\hat{v})]}_{=\varnothing} + \underbrace{E[(v-\hat{v})^2]}_{\to\varnothing} \approx E[x^2]$$

signal and noise are not correlated

Adaptative noise reduction - application



After cancelling mathernal ECG

Source: Widrow, B. ; Glover, J.R., Jr. ; McCool, J.M. ; Kaunitz, J. ;Williams, C.S. ; Hearn, R.H. ; Zeidler, J.R. ; Eugene Dong, Jr. ;Goodlin, R.C., **Adaptive noise cancelling: Principles and applications**, Proceedings of the IEEE Volume: 63 , Issue: 12, 1975

Adaptative filtering application

Adaptative filters are mainly applied to filtering of nonstationary signals, eg:

- In reduction of noise from the mains network and from electrical surgical instruments (f~120 Hz)
- In reduction of the energy of the ECG signal of a mother during registration of ECG signal of a fetus
- As a prediction model of biological signals in detection of their disturbances (eg. Ventricular fibrillation detection → implantation of defibrillators)

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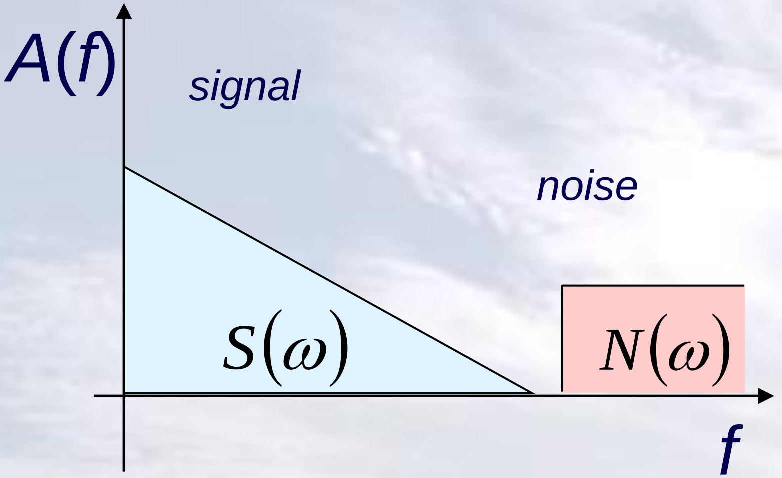
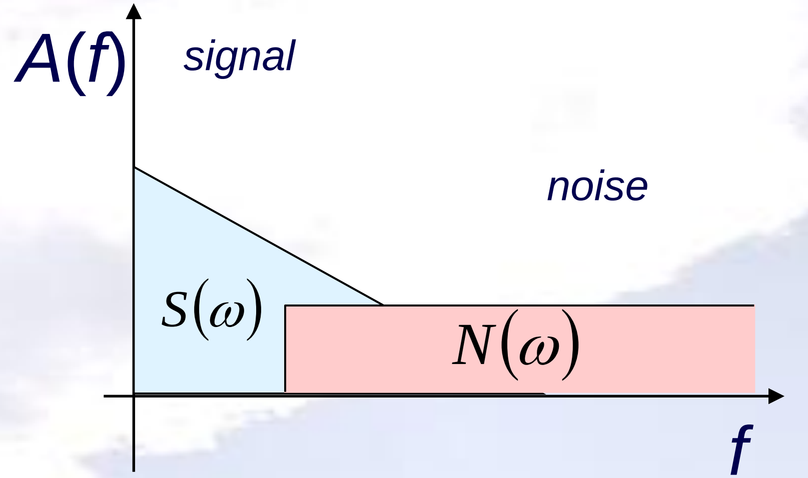
Synchronous averaging of the signal

Lowpass filters are not effective for smoothing the signals in which the noise frequency band overlaps the significant frequency components of the signal.

$$x(t) = s(t) + n(t)$$

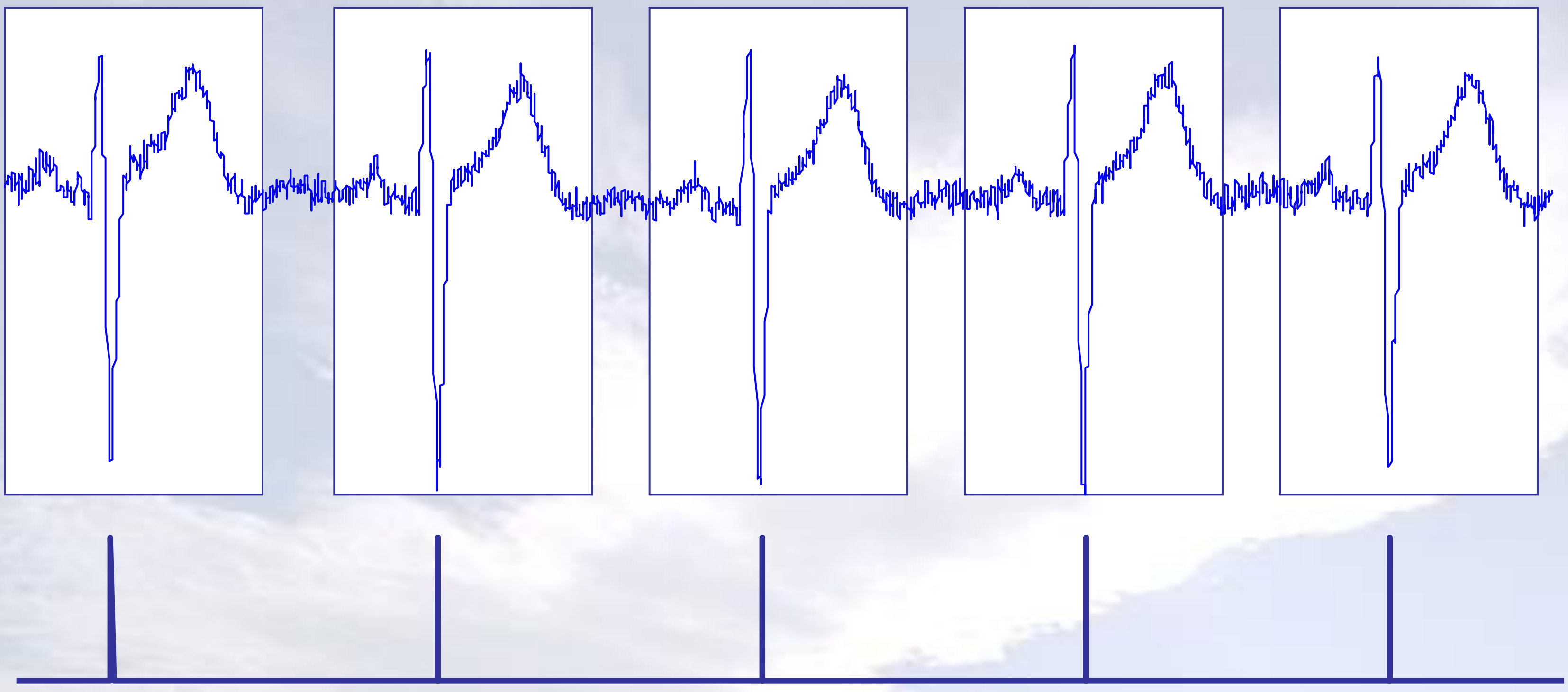




Frequency spectra of the signal and noise

Synchronous averaging of the signal

Idea of synchronious averaging



Synchronizing signal

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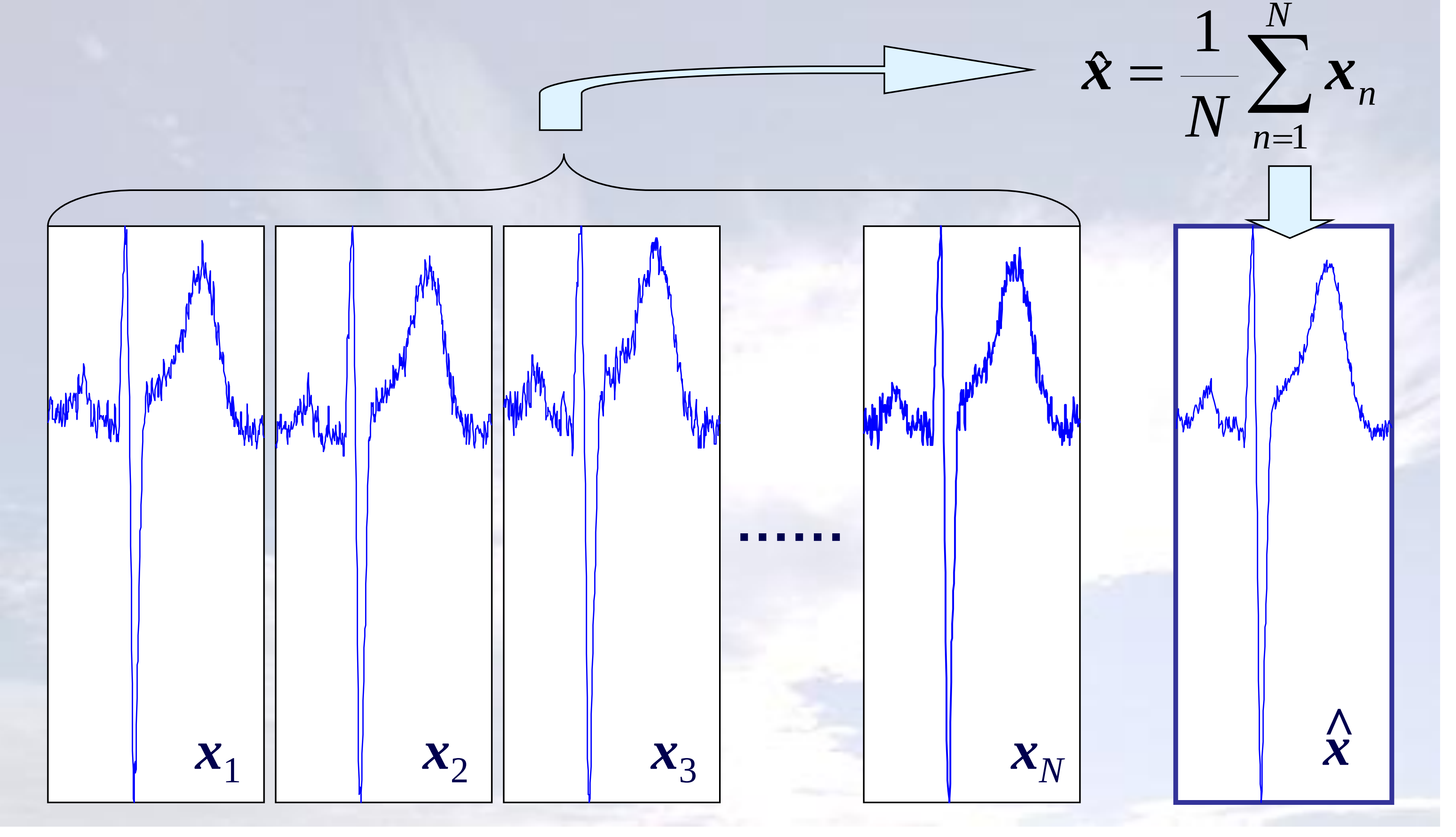
Synchronous averaging of the signal

Synchronious averaging of the signal is efficient under the following conditions:

- deterministic components of the signal should occur periodically (not necessarily at regularly spaced intervals)
- distrubance signal should be a random signal, uncorrelated with the deterministic components of the signal.
- there should be a possibility to detect signal features necessary for synchronization of the successive cycles.

Synchronous averaging of the signal



Synchronous averaging of the signal

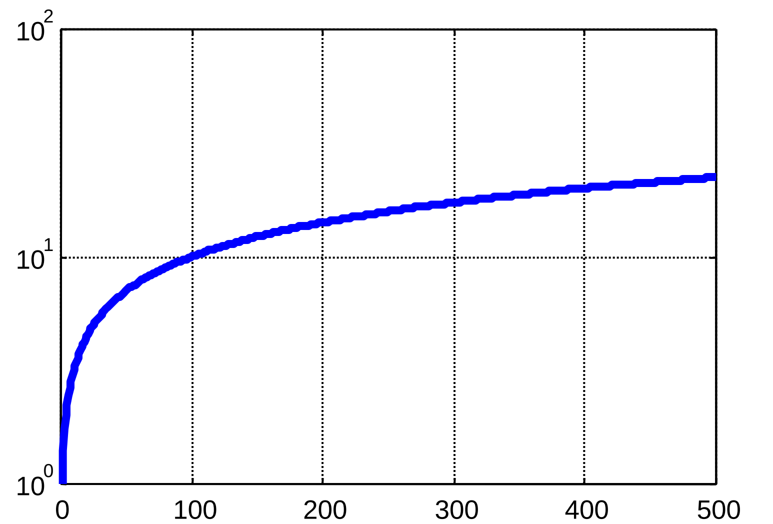
Standard deviation of the signal: σ_s

Standard deviation of noise: σ_n

Signal to noise ratio: $$SNR = \frac{\sigma_s}{\sigma_n}$$

After *N* averagings:

$$\boxed{SNR_N = \sqrt{N}\,\frac{\sigma_s}{\sigma_n}}$$



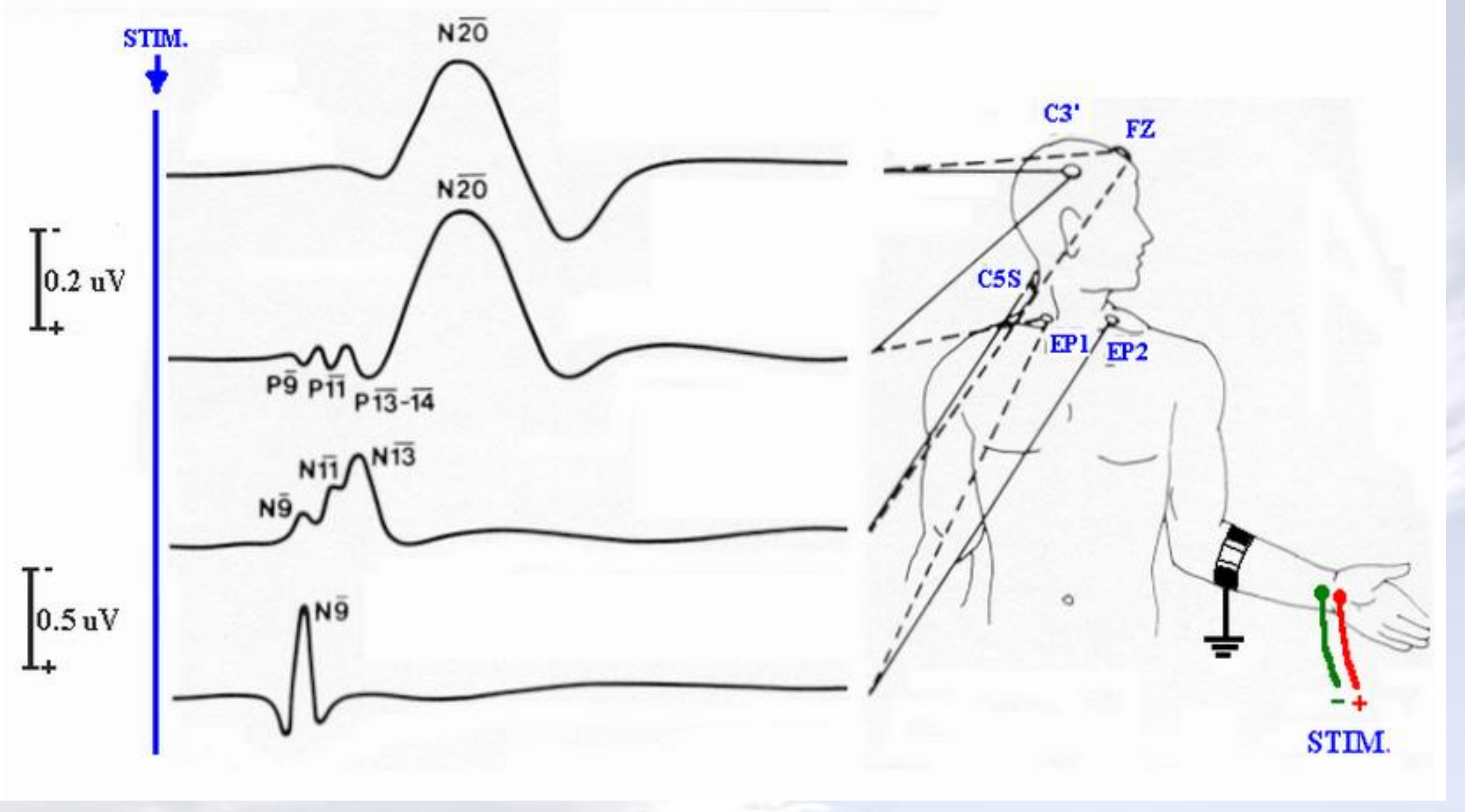
SNR impovement after *N* averagings: $\boxed{\sqrt{N}}$

Synchronous averaging of the signal

Applications:

- sub-noise signal detection ($\sigma_s << \sigma_n$) (telecommunications)
- EEG electric evoked potentials analysis, ie. Analysis of potentials of the **several microvolts** amplitude, generated within the brain due to periodic stimulation by: light (**visual evoked** potentials), sound (**auditory evoked** potentials) or touch (**touch-evoked** potentials).

Touch-evoked potentials



M. F. El-Bab, COGNITIVE EVENT RELATED POTENTIALS DURING A LEARNING TASK, PhD, University of Southampton, UK.

Median filtering

Median – the middle element of the orderly sequence, eg.:

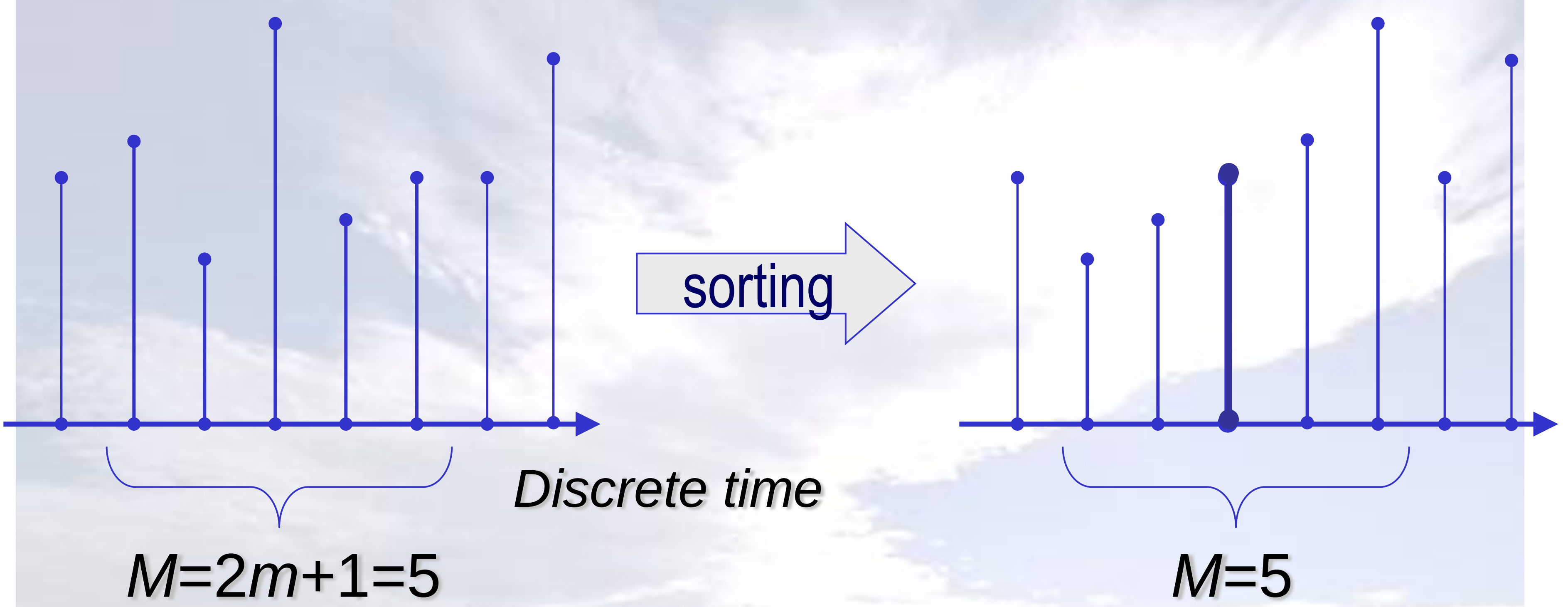
$$x(n)=\{1, 5, -7, 101, -25, 3, 0, 11, 7\}$$

Sequence sorting:

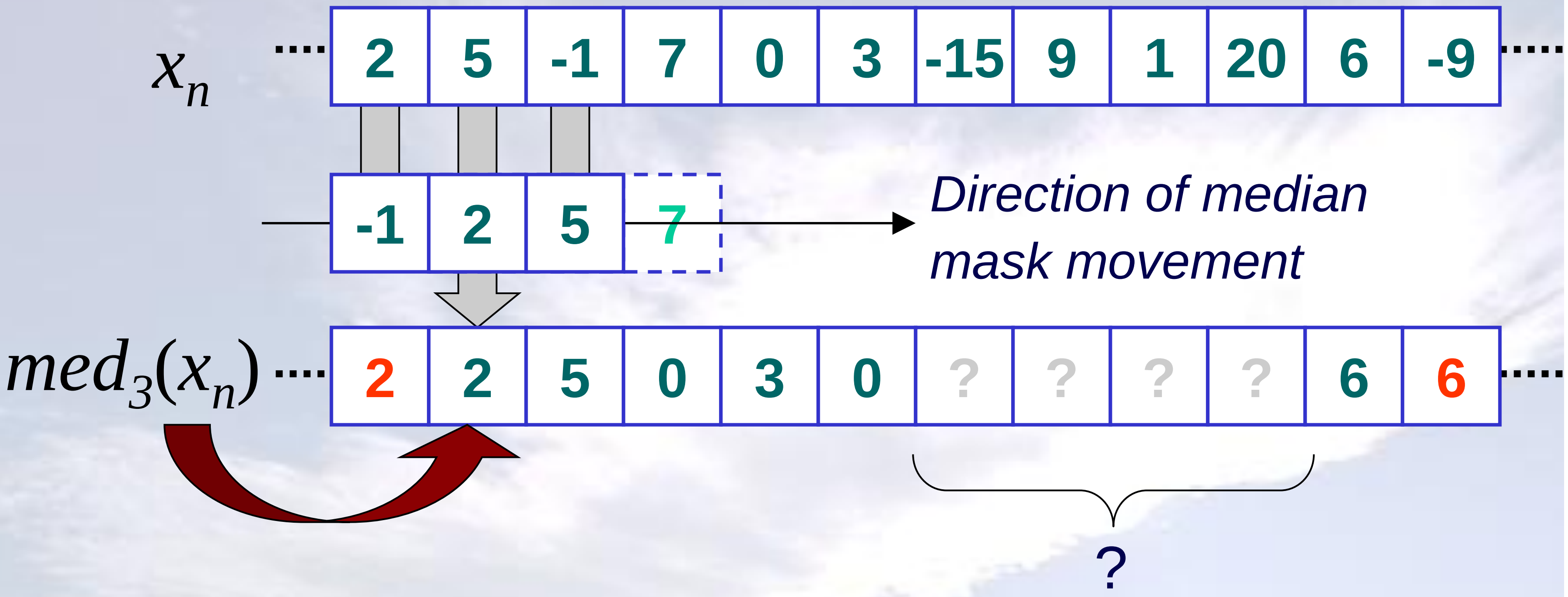
$$x_s(n)=\{-25, -7, 0, 1, \mathbf{3}, 5, 7, 11, 101\}$$

Middle element

Median filtering of signals



Median filtering of signals



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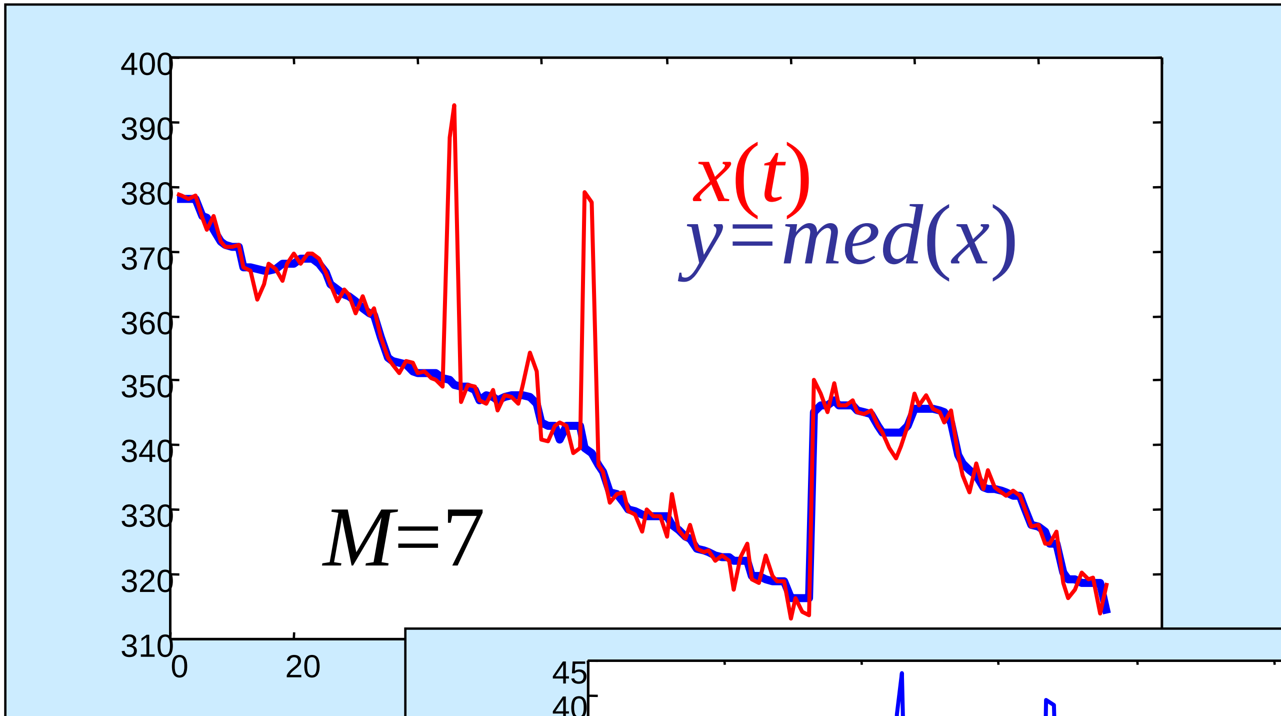
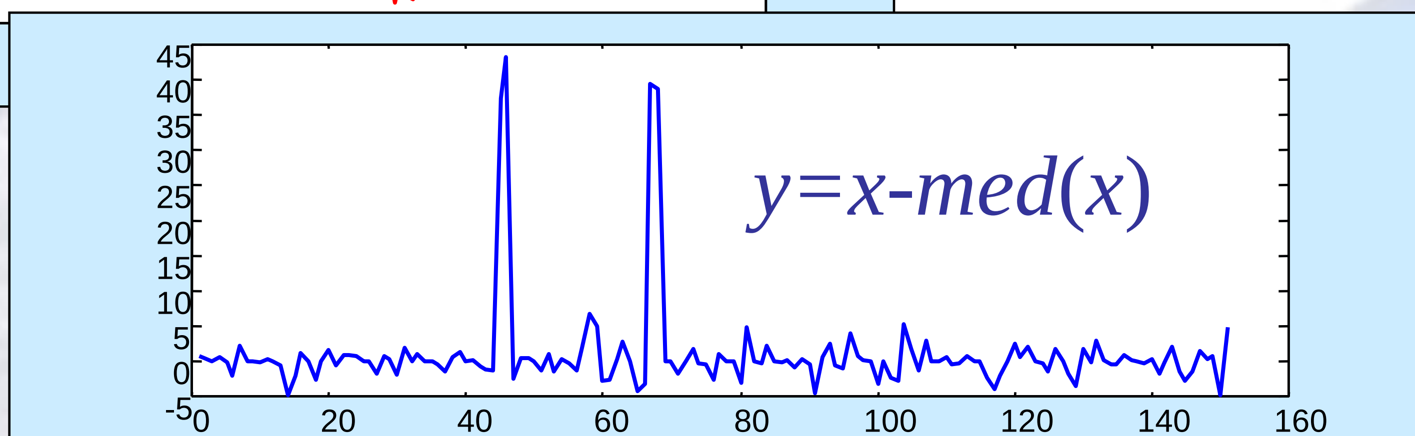
Median filtering - example





Digital filters - summary

1. Idea of filtering
2. Frequency characteristic of a filter
3. Ideal filters
4. Implementable filters
5. Difference equation
6. FIR and IIR filters
7. Linear phase and phase distortion
8. Filter design and signal filtering
9. Adaptative filtering
10. Median filtering ($\rightarrow$nonlinear filtering)

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„SIGNAL PROCESSING”

Prezentacja multimedialna współfinansowana przez Unię Europejską w ramach Europejskiego Funduszu Społecznego w projekcie pt.

„Innowacyjna dydaktyka bez ograniczeń - zintegrowany rozwój Politechniki Łódzkiej - zarządzanie Uczelnią, nowoczesna oferta edukacyjna i wzmacniania zdolności do zatrudniania osób niepełnosprawnych”